Integration

The Int Command

For example

$$\int x\,dx = \frac{1}{2}x^2$$

By matlab :

```
>> int('x')
```

MATLAB returns:

```
ans =
1/2*x^2
```

MATLAB can generate integrals that are entirely symbolic. That is, instead of:

```
>> int('x^2')

ans =

1/3*x^3
```

Consider:

```
>> int('x^n')

ans =

x^(n+1)/(n+1)
```

For example, we can define a trig function:

```
>> g = 'sin(n*t)';
```

If we just pass this function to int, it assumes that t is the integration variable:

```
>> int(g)

ans =

–1/n*cos(n*t)
```

However we can call int using the syntax int(f, v)

Where:
f : is the function to integrate
v : is the integration variable.

```
>> syms n

>> g = 'sin(n*t)';

>> int(g,n)

ans =

–1/t*cos(n*t)
```

EXAMPLE :

What is the integral of f(x) = bx? Evaluate the resulting expression for b = 2, x = 4.

SOLUTION:

We start by defining our symbolic variables:

```
>> syms b x
```

Now we define the function and integrate:

```
>> f = b^x;

>> F = int(f)

F =

1/log(b)*b^x
```

We can obtain a numerical value for the expression with the given values by calling subs. Enclose the variable list and substitution list in curly braces {}. In this case we write:

```
>> subs(F,{b,x},{2,4})
ans =
23.0831
```

EXAMPLE :
Compute ∫ x5 cos(x)dx .

SOLUTION :

With MATLAB, we can generate the answer on a single line:

```
>> F = int(x^5*cos(x))

F =

x^5*sin(x)+5*x^4*cos(x)–20*x^3*sin(x)60*x^2*cos(x)+120*cos(x)+120*x*sin(x)
```

We can use the command “pretty” to have MATLAB display the answer in a more pleasing format:

```
>> pretty(F)
```

x5 sin(x) + 5x4 cos(x) – 20x2 – 60x cos(x) + 120 cos(x) + 120x sin(x)

EXAMPLE :
Find ∫ 3y sec(x) dy .

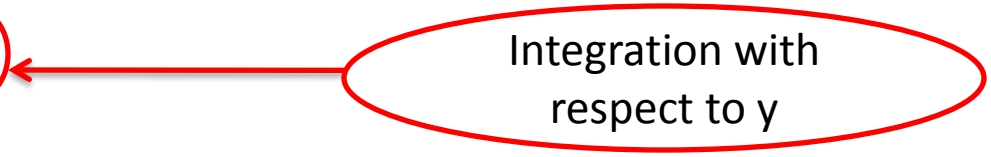
SOLUTION :

The integrand contains two variables, so we tell MATLAB that we want to integrate with respect to y:

```
>> int('3*y^2*sec(x)',y)

ans =

y^3*sec(x)
```



Had we wanted to integrate with respect to x instead we would have written:

```
>> int('3*y^2*sec(x)',x)

ans =

3*y^2*log(sec(x)+tan(x))
```

- ***The int command can be used for definite integration by passing the limits over which you want to calculate the integral.***

- ***If we enter int(f, a, b) then MATLAB integrates over the default independent variable and returns:***

For example:

$$\int_2^3 x\,dx = \frac{1}{2}x^2\Big|_2^3 = \frac{1}{2}(9-4) = \frac{5}{2}$$

would be calculated in MATLAB by writing:

```
>> int('x',2,3)

ans =

5/2
```

Continue

if we wanted MATLAB to generate the intermediate expression $\frac{1}{2}x^2$, *we could write:*

```
>> F = int('x')
F =
1/2*x^2

>> a = subs(F,3)–subs(F,2)
a =
2.5000
```

H.W

EXAMPLE 8-4

What is the area under the curve $f(x) = x^2 \cos x$ for $-6 \le x \le 6$?

EXAMPLE 8-5

Calculate $\int_0^{\infty} e^{-x^2} \sin x \, dx$.

Multidimensional Integration :

We can compute multidimensional integrals in MATLAB by using nested int statements. Suppose that we wanted to compute the indefinite integral:

$$\int\int\int x\, y^2 z^5 dx\, dy\, dz$$

This can be done with:

```
>> syms x y z

>> int(int(int(x*y^2*z^5,x),y),z)

ans =

1/36*x^2*y^3*z^6
```

➢Definite integration proceeds analogously. We can calculate:

$$\int_1^2 \int_2^4 x^2 y \, dx \, dy$$

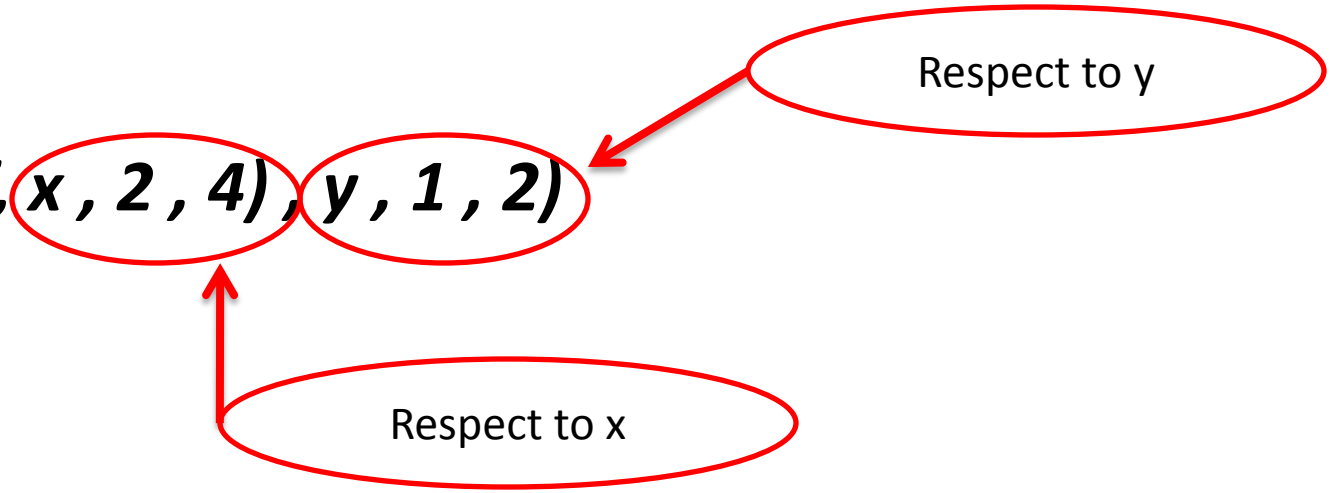
With the commands:

>> f = x^2*y;

>> int (int (f, x , 2 , 4) , y , 1 , 2)

ans =

28



EXAMPLE :

Find the volume of a cylinder of height h and radius a. What is the volume of a cylinder with radius a = 3.5 inches and height h = 5 inches?

Solution:

We will integrate using cylindrical coordinates with:

$$0 \le r \le a,\ 0 \le q \le 2p,\ 0 \le z \le h$$

The volume element in cylindrical coordinates is:

$$dV = r\,dr\,dqdz$$

So the volume of a cylinder of height h and radius a is:

Continue

$$V = \int_0^h \int_0^{2\pi} \int_0^a r \, dr \, d\theta \, dz$$

The commands to implement this in MATLAB are:

```
>> syms r theta z h a
>> V = int(int(int( r, r,0,a) , theta,0,2*pi) , z,0,h)
V =
a^2*pi*h
```

Function | Respect to r | Respect to theta | Respect to h

The volume of a cylinder with radius a = 3.5 inches and height h = 5 inches is:

```
>> subs(V,{a,h},{3.5,5})
ans =
192.4226
```

The answer is expressed in cubic inches.