What is calculus ?

Calculus is a study of change.

Also we can define calculus as the part of mathematics that deals with limits.

Calculus was first created to meet the mathematical needs of the scientists of the 17^{th} century.

In 17^{th} century , Isaac Newton invented his version of calculus in order to explain the motion of the planets around the sum.

Today, calculus is used in :

- Calculating the orbits of satellites and space craft's.
- Predicting population size.
- Forecasting the weather.
- Forecasting the global trends (economy).
- Designing x-ray and ultra-sound equipments.

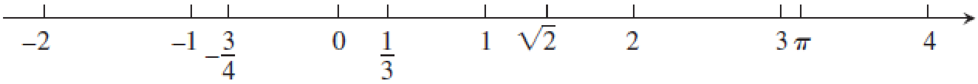
This list is practically endless.

In engineering , calculus has wide applications . Briefly , every field today uses calculus in some way.

Chapter One : Preliminaries

1.1. Real Numbers and the Real Line :

The real numbers can be represented geometrically as points on a number line called the **real line**.



The symbol $\mathbb{R}$ denotes either the real number system or, equivalently, the real line.

Rules for Inequalities

If a, b, and c are real numbers, then:

1. $a < b \Rightarrow a + c < b + c$
2. $a < b \Rightarrow a - c < b - c$
3. $a < b$ and $c > 0 \Rightarrow ac < bc$
4. $a < b$ and $c < 0 \Rightarrow bc < ac$
   Special case: $a < b \Rightarrow -b < -a$
5. $a > 0 \Rightarrow \frac{1}{a} > 0$
6. If a and b are both positive or both negative, then $a < b \Rightarrow \frac{1}{b} < \frac{1}{a}$

Intervals:

TABLE 1.1 Types of intervals

	Notation	Set description	Type	Picture
Finite:	(a, b)	$\{x \mid a < x < b\}$	Open	
	$[a, b]$	$\{x \mid a \leq x \leq b\}$	Closed	
	$[a, b)$	$\{x \mid a \leq x < b\}$	Half-open	
	$(a, b]$	$\{x \mid a < x \leq b\}$	Half-open	
Infinite:	(a, ∞)	$\{x \mid x > a\}$	Open	
	$[a, \infty)$	$\{x \mid x \geq a\}$	Closed	
	$(-\infty, b)$	$\{x \mid x < b\}$	Open	
	$(-\infty, b]$	$\{x \mid x \leq b\}$	Closed	
	$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	Both open and closed	

EXAMPLE 1 Solve the following inequalities and show their solution sets on the real line.

(a) $2x - 1 < x + 3$ **(b)** $-\frac{x}{3} < 2x + 1$ **(c)** $\frac{6}{x - 1} \geq 5$

Solution

(a)

$$2x - 1 < x + 3$$

$$2x < x + 4 \qquad \text{Add 1 to both sides.}$$

$$x < 4 \qquad \text{Subtract } x \text{ from both sides.}$$

The solution set is the open interval $(-\infty, 4)$ (Figure 1.1a).

(b)

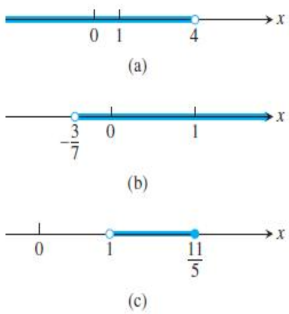
$$-\frac{x}{3} < 2x + 1$$

$$-x < 6x + 3 \qquad \text{Multiply both sides by 3.}$$

$$0 < 7x + 3 \qquad \text{Add } x \text{ to both sides.}$$

$$-3 < 7x \qquad \text{Subtract 3 from both sides.}$$

$$-\frac{3}{7} < x \qquad \text{Divide by 7.}$$



Solution sets for the inequalities in Example 1.