

Tutorial Examples :-

Ex.1:

$$\begin{aligned} \text{(a)} \quad y &= (3 - x^2)(x^3 - x + 1) \Rightarrow y' = (3 - x^2) \cdot \frac{d}{dx}(x^3 - x + 1) + (x^3 - x + 1) \cdot \frac{d}{dx}(3 - x^2) \\ &= (3 - x^2)(3x^2 - 1) + (x^3 - x + 1)(-2x) = -5x^4 + 12x^2 - 2x - 3 \\ \text{(b)} \quad y &= -x^5 + 4x^3 - x^2 - 3x + 3 \Rightarrow y' = -5x^4 + 12x^2 - 2x - 3 \end{aligned}$$

Ex.2:

$$y = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x} + 1\right)$$

$$\text{(a)} \quad y' = \left(x + x^{-1}\right) \cdot \left(1 + x^{-2}\right) + \left(x - x^{-1} + 1\right) \left(1 - x^{-2}\right) = 2x + 1 - \frac{1}{x^2} + \frac{2}{x^3}$$

$$\text{(b)} \quad y = x^2 + x + \frac{1}{x} - \frac{1}{x^2} \Rightarrow y' = 2x + 1 - \frac{1}{x^2} + \frac{2}{x^3}$$

Ex.3:

$$s = \tan t - t \Rightarrow \frac{ds}{dt} = \frac{d}{dt}(\tan t) - 1 = \sec^2 t - 1 = \tan^2 t$$

$$s = t^2 - \sec t + 1 \Rightarrow \frac{ds}{dt} = 2t - \frac{d}{dt}(\sec t) = 2t - \sec t \tan t$$

Ex.4

$$x^2(x - y)^2 = x^2 - y^2:$$

$$\text{Step 1:} \quad x^2 \left[2(x - y) \left(1 - \frac{dy}{dx} \right) \right] + (x - y)^2(2x) = 2x - 2y \frac{dy}{dx}$$

$$\text{Step 2:} \quad -2x^2(x - y) \frac{dy}{dx} + 2y \frac{dy}{dx} = 2x - 2x^2(x - y) - 2x(x - y)^2$$

$$\text{Step 3:} \quad \frac{dy}{dx} [-2x^2(x - y) + 2y] = 2x [1 - x(x - y) - (x - y)^2]$$

$$\begin{aligned} \text{Step 4:} \quad \frac{dy}{dx} &= \frac{2x [1 - x(x - y) - (x - y)^2]}{-2x^2(x - y) + 2y} = \frac{x [1 - x(x - y) - (x - y)^2]}{y - x^2(x - y)} = \frac{x (1 - x^2 + xy - x^2 + 2xy - y^2)}{x^2y - x^3 + y} \\ &= \frac{x - 2x^3 + 3x^2y - xy^2}{x^2y - x^3 + y} \end{aligned}$$

Ex.5:

$$x + \sin y = xy \Rightarrow 1 + (\cos y) \frac{dy}{dx} = y + x \frac{dy}{dx} \Rightarrow (\cos y - x) \frac{dy}{dx} = y - 1 \Rightarrow \frac{dy}{dx} = \frac{y - 1}{\cos y - x}$$

Ex.6: Two cars leave a depot . Car A traveling east at 40 mph and car B traveling north at 30 mph . How fast is the distance between the cars changing after 6 minutes ?