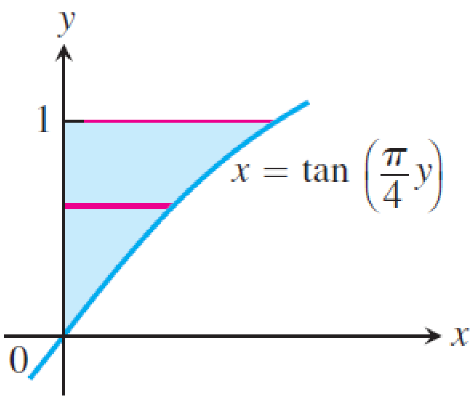
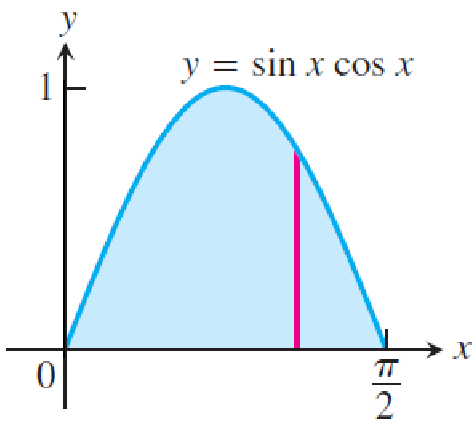
Ex.9: Find the volume of the solid generated by revolving the shaded region about the given axis :

15. About the y-axis



16. About the x-axis



Sol./

15. $R(x) = \tan\left(\frac{\pi}{4}y\right)$; $u = \frac{\pi}{4}y \Rightarrow du = \frac{\pi}{4}dy \Rightarrow 4\,du = \pi\,dy$; $y = 0 \Rightarrow u = 0$, $y = 1 \Rightarrow u = \frac{\pi}{4}$;

$$V = \int_0^1 \pi[R(y)]^2\,dy = \pi\int_0^1 \left[\tan\left(\frac{\pi}{4}y\right)\right]^2 dy = 4\int_0^{\pi/4} \tan^2 u\,du = 4\int_0^{\pi/4}(-1+\sec^2 u)\,du = 4[-u+\tan u]_0^{\pi/4}$$

$$= 4\left(-\frac{\pi}{4}+1-0\right) = 4-\pi$$

16. $R(x) = \sin x \cos x$; $R(x) = 0 \Rightarrow a = 0$ and $b = \frac{\pi}{2}$ are the limits of integration; $V = \int_0^{\pi/2} \pi[R(x)]^2\,dx$

$$= \pi\int_0^{\pi/2} (\sin x \cos x)^2\,dx = \pi\int_0^{\pi/2} \frac{(\sin 2x)^2}{4}\,dx;\ \left[u = 2x \Rightarrow du = 2\,dx \Rightarrow \frac{du}{8} = \frac{dx}{4};\ x = 0 \Rightarrow u = 0,\right.$$

$$\left.x = \frac{\pi}{2} \Rightarrow u = \pi\right] \rightarrow V = \pi\int_0^{\pi} \frac{1}{8}\sin^2 u\,du = \frac{\pi}{8}\left[\frac{u}{2}-\frac{1}{4}\sin 2u\right]_0^{\pi} = \frac{\pi}{8}\left[\left(\frac{\pi}{2}-0\right)-0\right] = \frac{\pi^2}{16}$$

Ex.10:

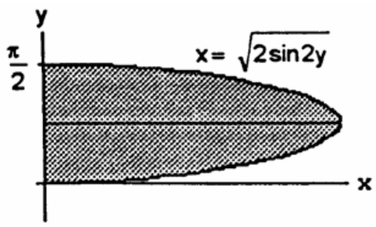
27. The region enclosed by $x = \sqrt{2\sin 2y}$, $0 \le y \le \pi/2$, $x = 0$

Sol./

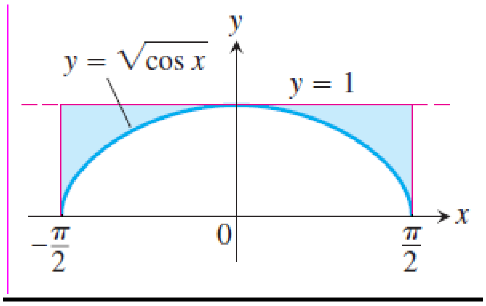
27. $R(y) = \sqrt{2\sin 2y} \Rightarrow V = \int_0^{\pi/2} \pi[R(y)]^2\,dy$

$$= \pi\int_0^{\pi/2} 2\sin 2y\,dy = \pi[-\cos 2y]_0^{\pi/2}$$

$$= \pi[1-(-1)] = 2\pi$$



Ex.11: Find the volume of the solid generated by revolving the shaded region about the x-axis .

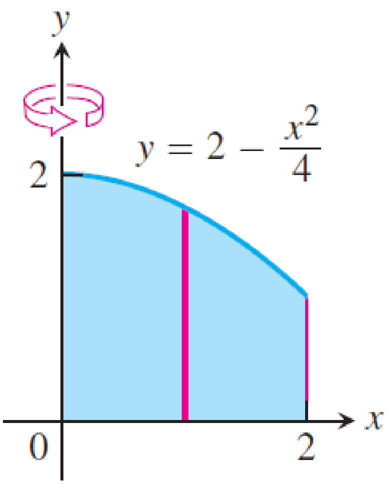


Sol./

For the sketch given, $a = -\frac{\pi}{2}$, $b = \frac{\pi}{2}$; $R(x) = 1$, $r(x) = \sqrt{\cos x}$; $V = \int_a^b \pi \left([R(x)]^2 - [r(x)]^2\right) dx$

$$= \int_{-\pi/2}^{\pi/2} \pi(1 - \cos x)\, dx = 2\pi \int_0^{\pi/2} (1 - \cos x)\, dx = 2\pi[x - \sin x]_0^{\pi/2} = 2\pi\left(\frac{\pi}{2} - 1\right) = \pi^2 - 2\pi$$

Ex.12: Find the volume of the solid generated by revolving the shaded region about the given axis :



Sol./

For the sketch given, a = 0, b = 2;

$$V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx = \int_0^2 2\pi x \left(2 - \frac{x^2}{4}\right) dx = 2\pi \int_0^2 \left(2x - \frac{x^3}{4}\right) dx = 2\pi \left[x^2 - \frac{x^4}{16}\right]_0^2 = 2\pi(4-1) = 6\pi$$

Ex.13: Find the volume of the solid generated by revolving the shaded region about the y- axis :

$$y = 2 - x^2, \quad y = x^2, \quad x = 0$$

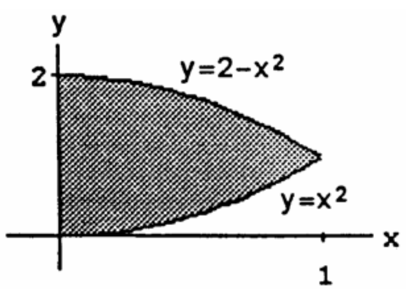
Sol./

a = 0, b = 1;

$$V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx = \int_0^1 2\pi x\,[(2 - x^2) - x^2]\,dx$$

$$= 2\pi \int_0^1 x\,(2 - 2x^2)\,dx = 4\pi \int_0^1 (x - x^3)\,dx$$

$$= 4\pi \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 4\pi\left(\frac{1}{2} - \frac{1}{4}\right) = \pi$$



Ex.14: Find the volume of the solid generated by revolving the shaded region about the x- axis :

$$x = \sqrt{y}, \quad x = -y, \quad y = 2$$

Sol./

$$c = 0,\ d = 2;$$

$$V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^2 2\pi y\left[\sqrt{y} - (-y)\right] dy$$

$$= 2\pi \int_0^2 \left(y^{3/2} + y^2\right) dy = 2\pi\left[\frac{2y^{5/2}}{5} + \frac{y^3}{3}\right]_0^2$$

$$= 2\pi\left[\frac{2}{5}\left(\sqrt{2}\right)^5 + \frac{2^3}{3}\right] = 2\pi\left(\frac{8\sqrt{2}}{5} + \frac{8}{3}\right) = 16\pi\left(\frac{\sqrt{2}}{5} + \frac{1}{3}\right)$$

$$= \frac{16\pi}{15}\left(3\sqrt{2} + 5\right)$$