

A Review of Trigonometric Functions:

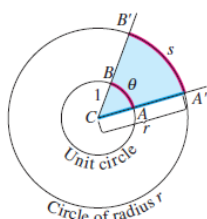


FIGURE The radian measure of angle ACB is the length θ of arc AB on the unit circle centered at C . The value of θ can be found from any other circle, however, as the ratio s/r . Thus $s = r\theta$ is the length of arc on a circle of radius r when θ is measured in radians.

Radian Measure

In navigation and astronomy, angles are measured in degrees, but in calculus it is best to use units called *radians* because of the way they simplify later calculations.

The **radian measure** of the angle ACB at the center of the unit circle (Figure 1.63) equals the length of the arc that ACB cuts from the unit circle. Figure 1.63 shows that $s = r\theta$ is the **length of arc** cut from a circle of radius r when the subtending angle θ producing the arc is measured in radians.

Since the circumference of the circle is 2π and one complete revolution of a circle is 360° , the relation between radians and degrees is given by

$$\pi \text{ radians} = 180^\circ.$$

For example, 45° in radian measure is

$$45 \cdot \frac{\pi}{180} = \frac{\pi}{4} \text{ rad},$$

and $\pi/6$ radians is

$$\frac{\pi}{6} \cdot \frac{180}{\pi} = 30^\circ.$$

Conversion Formulas

$$1 \text{ degree} = \frac{\pi}{180} (\approx 0.02) \text{ radians}$$

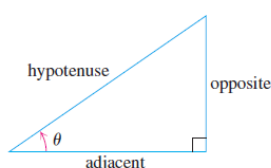
$$\text{Degrees to radians: multiply by } \frac{\pi}{180}$$

$$1 \text{ radian} = \frac{180}{\pi} (\approx 57) \text{ degrees}$$

$$\text{Radians to degrees: multiply by } \frac{180}{\pi}$$

Figure 1.64 shows the angles of two common triangles in both measures.

An angle in the xy -plane is said to be in **standard position** if its vertex lies at the origin and its initial ray lies along the positive x -axis (Figure 1.64). Angles measured counter-clockwise from the positive x -axis are assigned positive measures; angles measured clock-



$$\begin{aligned} \sin \theta &= \frac{\text{opp}}{\text{hyp}} & \csc \theta &= \frac{\text{hyp}}{\text{opp}} \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}} & \sec \theta &= \frac{\text{hyp}}{\text{adj}} \\ \tan \theta &= \frac{\text{opp}}{\text{adj}} & \cot \theta &= \frac{\text{adj}}{\text{opp}} \end{aligned}$$

FIGURE Trigonometric ratios of an acute angle.

The Six Basic Trigonometric Functions

You are probably familiar with defining the trigonometric functions of an acute angle in terms of the sides of a right triangle (Figure 1.64). We extend this definition to obtuse and negative angles by first placing the angle in standard position in a circle of radius r . We then define the trigonometric functions in terms of the coordinates of the point $P(x, y)$ where the angle's terminal ray intersects the circle (Figure 1.65).

$$\begin{aligned} \text{sine: } \sin \theta &= \frac{y}{r} & \text{cosecant: } \csc \theta &= \frac{r}{y} \\ \text{cosine: } \cos \theta &= \frac{x}{r} & \text{secant: } \sec \theta &= \frac{r}{x} \\ \text{tangent: } \tan \theta &= \frac{y}{x} & \text{cotangent: } \cot \theta &= \frac{x}{y} \end{aligned}$$

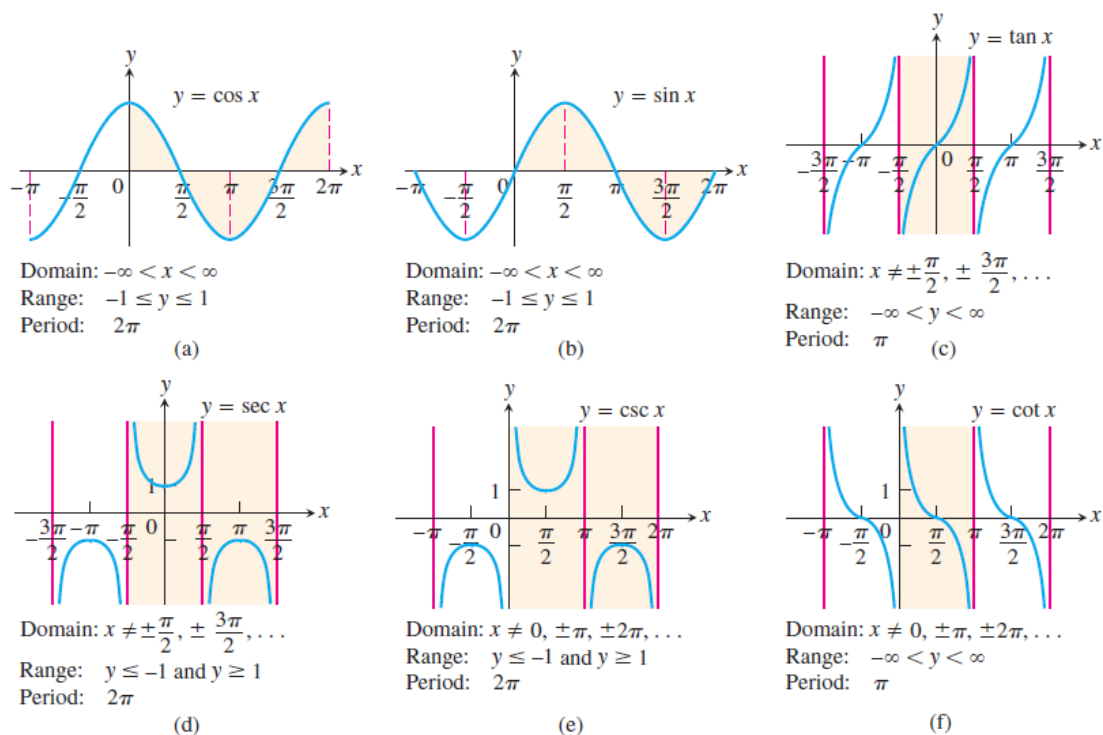


FIGURE Graphs of the (a) cosine, (b) sine, (c) tangent, (d) secant, (e) cosecant, and (f) cotangent functions using radian measure. The shading for each trigonometric function indicates its periodicity.