Chapter Nine : Matrices, Determinants and Cramer's Rule

Matrices : Matrices have many applications in science and engineering.

A rectangular array of numbers like $A = \begin{bmatrix} 4 & 2 & 1 \\ -2 & 0 & 3 \end{bmatrix}$ is called a matrix.

Matrix A has 2 rows and 3 columns, then A is called 2 by 3 matrix.

Each number is called element and denoted by (a_{ij}) , where i : no. of row and

j : no. of column.

For example : $a_{11} = 4$, $a_{23} = 3$, $a_{13} = 1$, ….

If the number of row equals to the number of column then the matrix is called square matrix.

Equal matrices have identical corresponding elements .

For example : $\begin{bmatrix} 2 & 4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & -2 \end{bmatrix}$

Ex. Find x , y and z :

$$\begin{bmatrix} x \\ x+2 \\ 2y-3 \end{bmatrix} = \begin{bmatrix} 4 \\ y \\ z \end{bmatrix}$$

Sol.

$x = 4$

$x + 2 = y \rightarrow y = 6$

$2y - 3 = z \rightarrow z = 9$.

Diagonal Matrix: Is a square matrix where all the element are zeroes except the principal (or main) diagonal.

For example : $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Identity Matrix (I) : Is a diagonal matrix with the element of principle diagonal are ones.

For example : $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 2×2 identity matrix.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 3×3 identity matrix.

Addition and Subtraction of matrices:

Ex.1/ (Addition) : $\begin{bmatrix} 8 & 3 & 4 \\ 0 & -1 & 9 \end{bmatrix} + \begin{bmatrix} 5 & -2 & 1 \\ 6 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 13 & 1 & 5 \\ 6 & 2 & 14 \end{bmatrix}$

Ex.2/ (Subtraction) : $\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix}$

Multiplication a matrix by a constant:

Ex. $A = \begin{bmatrix} 3 & 1 \\ 7 & -1 \\ 2 & 8 \end{bmatrix} \rightarrow 3A = \begin{bmatrix} 9 & 3 \\ 21 & -3 \\ 6 & 24 \end{bmatrix}$

Multiplication of two matrices:

We can only multiply two matrices if the number of columns in first matrix equals to the number of rows in second matrix.

The resulting matrix has number of rows same as first matrix . and number of columns same as second matrix.

For example : [2×3 matrix]* [3×4 matrix] = [2×4 matrix].

How to multiply tow matrices ?

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} * \begin{bmatrix} u & v \\ w & x \\ y & z \end{bmatrix} = \begin{bmatrix} au + bw + cy & av + bx + cz \\ du + ew + fy & dv + ex + fz \end{bmatrix}$$

Ex.1:

$$\begin{bmatrix} 0 & -1 & 2 \\ 4 & 11 & 2 \end{bmatrix} * \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 0(3) - 1(1) + 2(6) & 0(1) - 1(2) + 2(1) \\ 4(3) + 11(1) + 2(6) & 4(-1) + 11(2) + 2(1) \end{bmatrix} =$$

$$\begin{bmatrix} 11 & 0 \\ 35 & 20 \end{bmatrix}$$

Notes: 1) If A is a matrix then A*I = A.

2) If A and B are matrices then A*B may be equals to B*A.

Ex.2: If possible , find AB and BA .

$$A = \begin{bmatrix} -2 & 1 & 7 \\ 3 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix} , \quad B = \begin{bmatrix} 4 & -1 & 5 \end{bmatrix}$$

Sol.:

AB not possible .

BA = [4(-2) – 1(3) + 5(0)   4(1) – 1(-1) + 5(2)   4(7) – 1(0) +5(-1)]

BA = [-11   15   23]

Ex.3: Find : $\begin{bmatrix} cos60 & -sin60 & 0 \\ sin60 & cos60 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$ (H.W)