4) For determinants of any size :

This method called (expansion by minors) :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} \quad \text{(from 1}^{\text{st}} \text{ row)}$$

$$\text{Or} = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} \quad \text{(from 2}^{\text{nd}} \text{ row)}$$

Or from any row or column.

Note : To simplify the calculation , take row or column with greater number of zeroes.

Ex. Find the determinant for the matrix:

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$$

Sol. Take 1st row :

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix} = (-1 * 1) - (3 * -2) = 5$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = -[(3 * 1) - (2 * -2)] = -7$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix} = [(3 * 3) - (2 * -1)] = 11$$

Det.A = $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

= 2(5)+1(-7)+3(11)

= 36 (same result with last example)

Transpose of a matrix (A^T) :

It is the matrix obtained from interchanging the rows by columns.

Ex. $A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -1 & 3 \\ 5 & 7 & 8 \end{bmatrix} \longrightarrow A^T = \begin{bmatrix} 3 & 2 & 5 \\ 4 & -1 & 7 \\ 1 & 3 & 8 \end{bmatrix}$

The inverse of a matrix (A^{-1}) :

The inverse matrix A^{-1} for the matrix A is the matrix in which $A*A^{-1} = I$.

We will study two methods to find A^{-1}.

Method 1: By using the determinant of a matrix (for 2×2 matrices only):

Ex. Find A-1 for $A = \begin{bmatrix} 2 & -3 \\ 4 & -7 \end{bmatrix}$

Sol. 1) Interchange main diagonal elements.:

$$\begin{bmatrix} -7 & -3 \\ 4 & 2 \end{bmatrix}$$

2) Change signs of the other two elements:

$$\begin{bmatrix} -7 & 3 \\ -4 & 2 \end{bmatrix}$$

3) Find det.A : (2*-7) – (4*-3) = -2

4) Multiply result from step 2 by 1/det.A to get the inverse of the matrix :

$$A^{-1} = 1/\text{det.A} \begin{bmatrix} -7 & 3 \\ -4 & 2 \end{bmatrix} = 1/{-2} \begin{bmatrix} -7 & 3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 3.5 & -1.5 \\ 2 & -1 \end{bmatrix}$$

For checking : $\begin{bmatrix} 2 & -3 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} 3.5 & -1.5 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ O.K.

Method 2 : By using adjoint matrix method (for matrix of any size):

$$A^{-1} = \frac{adj.A}{det.A}$$

The adjoint matrix is found by replacing each element in the matrix with its cofactor , and copy + & - signs as follows :

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

And then finding the transpose of the resulting matrix.