Ex.1: Find A^{-1} for $\begin{bmatrix} 5 & 6 & 1 \\ 0 & 3 & -3 \\ 4 & -7 & 2 \end{bmatrix}$

1) Replace elements with cofactor : and copy + & - signs.

$A_{11} = (3*2) - (-7 *-3) = -15$, $A_{12} = 12$, $A_{13} = -12$, $A_{21} = 19$, $A_{22} = 6$

$A_{23} = -59$, $A_{31} = -21$, $A_{32} = -15$, $A_{33} = 15$.

$$\Rightarrow \begin{bmatrix} +(-15) & -(12) & +(-12) \\ -(19) & +(6) & -(-59) \\ +(-21) & -(-15) & +(15) \end{bmatrix} = \begin{bmatrix} -15 & -12 & -12 \\ -19 & 6 & 59 \\ -21 & 15 & 15 \end{bmatrix}$$

2) Transpose the matrix obtained from step 1 to find adj.A :

$$\Rightarrow adj.A = \begin{bmatrix} -15 & -19 & -21 \\ -12 & 6 & 15 \\ -12 & 59 & 15 \end{bmatrix}$$

3) Find det.A :

$$det.A = \begin{bmatrix} 5 & 6 & 1 \\ 0 & 3 & -3 \\ 4 & -7 & 2 \end{bmatrix} = 5(-15) - 0(19) + 4(-21) = -159$$

4) Find A^{-1}:

$$A^{-1} = \frac{adj\ .A}{det\ .A} = \frac{1}{-159}\begin{bmatrix} -15 & -19 & -21 \\ -12 & 6 & 15 \\ -12 & 59 & 15 \end{bmatrix} = \begin{bmatrix} 0.094 & 0.119 & 0.132 \\ 0.075 & -0.038 & -0.094 \\ 0.075 & -0.371 & -0.094 \end{bmatrix}$$

Ex.2: Find A^{-1} for $\begin{bmatrix} -2 & 6 & 1 \\ 0 & 3 & -3 \\ 4 & -7 & 3 \end{bmatrix}$ (H.W).

Cramer's Rule :

Cramer's Rule is used to solve the simultaneous linear equations.

If we have the equations :

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

. . .

. . .

. . .

$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

$A = \begin{bmatrix} a_{11} & a_{12} \dots\dots\dots & a_{1n} \\ : & : & : \\ a_{n1} & a_{n2} & a_{nn} \end{bmatrix}$ Matrix of coefficients

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}$ Matrix of variables.

$b = \begin{bmatrix} b_1 \\ b_2 \\ b_n \end{bmatrix}$ Matrix of constants.

{A}{x} = {b}

$\boldsymbol{x_i} = \dfrac{\boldsymbol{det.A(with\ replacing\ the\ column\ i\ with\ b)}}{\boldsymbol{det.A}}$ Cramer's Rule