Ex.1: Solve 3x – y = 9

X + 2y = -4

Sol.:

$$det.A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} = 6 + 1 = 7$$

$$x = \frac{\begin{bmatrix} 9 & -1 \\ -4 & 2 \end{bmatrix}}{7} = \frac{18 - 4}{7} = \frac{14}{7} = 2$$

$$y = \frac{\begin{bmatrix} 3 & 9 \\ 1 & -4 \end{bmatrix}}{7} = \frac{-12 - 9}{7} = \frac{-21}{7} = -3$$

Values of (x & y) must satisfy the two equations.

Ex.2: solve :

$$2x + 3y + z = 2$$

$$-x + 2y + 3z = -1$$

$$-3x - 3y + z = 0$$

Sol.:

$$det.A = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ -3 & -3 & 1 \end{bmatrix} = 2(11) + 1(6) - 3(7) = 7$$

$$x = \frac{\begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 0 & -3 & 1 \end{bmatrix}}{7} = \frac{2(11) + 1(6)}{7} = \frac{28}{7} = 4$$

$$y = \frac{\begin{bmatrix} 2 & 2 & 1 \\ -1 & -1 & 3 \\ -3 & 0 & 1 \end{bmatrix}}{7} = \frac{2(-1) + 1(2) - 3(7)}{7} = \frac{-21}{7} = -3$$

$$z = \frac{\begin{bmatrix} 2 & 3 & 2 \\ -1 & 2 & -1 \\ -3 & -3 & 0 \end{bmatrix}}{7} = \frac{2(9)+1(3)}{7} = \frac{21}{7} = 3$$

Values of (x , y & z) must satisfy the three equations.

Useful facts about determinants :

1) If two rows (or columns) of a matrix are identity, the determinant is zero.

Ex.: $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & -1 & 5 \\ 2 & 1 & 4 \end{bmatrix} \Rightarrow$ det.A = 0 (because 1^{st} & 3^{rd} row are identical).

2) Interchanging two rows (or column) of a matrix changes the sign of the determinant.

Ex.: $A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix} \Rightarrow$ det.A = (2*-1) – (3*1) = 5

$B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \Rightarrow$ det.B = 5 (1^{st} column with 2^{nd} column)

3) det.A^T = det.A

4) Multiplying each element of a row (or column) constant C multiplies the determinant by C.

Ex.: $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \longrightarrow B = \begin{bmatrix} 3(1) & 3(2) \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 3 & -1 \end{bmatrix}$

det.A = -7 , det.B = 3*-7 = -21

5) If all elements of a matrix above (or below) the main diagonal are zeroes , then the determinant of the matrix is the product of the elements on the main diagonal.

Ex.: $A = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$, det.A = 3*1*2 = 6