

2.2. Continuity :

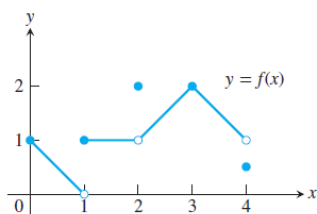


FIGURE The function is continuous on $[0, 4]$ except at $x = 1$, $x = 2$, and $x = 4$

Continuity at a Point

To understand continuity, we need to consider a function like the one in Figure whose limits we investigated

EXAMPLE Investigating Continuity

Find the points at which the function f in Figure is continuous and the points at which f is discontinuous.

Solution The function f is continuous at every point in its domain $[0, 4]$ except at $x = 1$, $x = 2$, and $x = 4$. At these points, there are breaks in the graph. Note the relationship between the limit of f and the value of f at each point of the function's domain.

Points at which f is continuous:

$$\begin{aligned} \text{At } x = 0, & \quad \lim_{x \rightarrow 0^+} f(x) = f(0). \\ \text{At } x = 3, & \quad \lim_{x \rightarrow 3} f(x) = f(3). \\ \text{At } 0 < c < 4, c \neq 1, 2, & \quad \lim_{x \rightarrow c} f(x) = f(c). \end{aligned}$$

Points at which f is discontinuous:

$$\begin{aligned} \text{At } x = 1, & \quad \lim_{x \rightarrow 1} f(x) \text{ does not exist.} \\ \text{At } x = 2, & \quad \lim_{x \rightarrow 2} f(x) = 1, \text{ but } 1 \neq f(2). \\ \text{At } x = 4, & \quad \lim_{x \rightarrow 4^-} f(x) = 1, \text{ but } 1 \neq f(4). \\ \text{At } c < 0, c > 4, & \quad \text{these points are not in the domain of } f. \end{aligned}$$

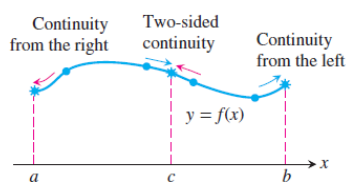


FIGURE Continuity at points a , b , and c .

To define continuity at a point in a function's domain, we need to define continuity at an interior point (which involves a two-sided limit) and continuity at an endpoint (which involves a one-sided limit)

DEFINITION Continuous at a Point

Interior point: A function $y = f(x)$ is **continuous at an interior point** c of its domain if

$$\lim_{x \rightarrow c} f(x) = f(c).$$

Endpoint: A function $y = f(x)$ is **continuous at a left endpoint** a or is **continuous at a right endpoint** b of its domain if

$$\lim_{x \rightarrow a^+} f(x) = f(a) \quad \text{or} \quad \lim_{x \rightarrow b^-} f(x) = f(b), \quad \text{respectively.}$$

Continuity Test

A function $f(x)$ is continuous at $x = c$ if and only if it meets the following three conditions.

1. $f(c)$ exists (c lies in the domain of f)
2. $\lim_{x \rightarrow c} f(x)$ exists (f has a limit as $x \rightarrow c$)
3. $\lim_{x \rightarrow c} f(x) = f(c)$ (the limit equals the function value)