Tutorial Examples :-

Ex.1 :

$\lim_{x \to 0} \frac{x}{|x|}$ does not exist because $\frac{x}{|x|} = \frac{x}{x} = 1$ if $x > 0$ and $\frac{x}{|x|} = \frac{x}{-x} = -1$ if $x < 0$. As x approaches 0 from the left, $\frac{x}{|x|}$ approaches -1. As x approaches 0 from the right, $\frac{x}{|x|}$ approaches 1. There is no single number L that all the function values get arbitrarily close to as $x \to 0$.

Ex.2 :

25. $\lim_{x \to -1} 3x(2x - 1) = 3(-1)(2(-1) - 1) = 9$

26. $\lim_{x \to -1} \frac{3x^2}{2x-1} = \frac{3(-1)^2}{2(-1)-1} = \frac{3}{-3} = -1$

27. $\lim_{x \to \frac{\pi}{2}} x \sin x = \frac{\pi}{2} \sin \frac{\pi}{2} = \frac{\pi}{2}$

28. $\lim_{x \to \pi} \frac{\cos x}{1-\pi} = \frac{\cos \pi}{1-\pi} = \frac{-1}{1-\pi} = \frac{1}{\pi-1}$

Ex.3 :

$$\lim_{t \to 1} \frac{t^2+t-2}{t^2-1} = \lim_{t \to 1} \frac{(t+2)(t-1)}{(t-1)(t+1)} = \lim_{t \to 1} \frac{t+2}{t+1} = \frac{1+2}{1+1} = \frac{3}{2}$$

Ex.4:

$$\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x \to 9} \frac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim_{x \to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

Ex.5 :

(a) $\lim_{t \to 0^+} \left[\frac{1}{t^{3/5}} + 7\right] = \infty$

(b) $\lim_{t \to 0^-} \left[\frac{1}{t^{3/5}} + 7\right] = -\infty$

Ex.6:

$$\lim_{x \to \pm\infty} \frac{3x+4}{2x-5} = \lim_{x \to \pm\infty} \frac{3+\frac{4}{x}}{2-\frac{5}{x}} = \lim_{t \to 0} \frac{3+4t}{2-5t} = \frac{3}{2}, \quad \left(t = \frac{1}{x}\right)$$

Ex.7:

$$\lim_{x \to \infty} \left(\frac{3}{x^2} - \cos\frac{1}{x}\right)\left(1 + \sin\frac{1}{x}\right) = \lim_{\theta \to 0^+} (3\theta^2 - \cos\theta)(1 + \sin\theta) = (0 - 1)(1 + 0) = -1, \quad \left(\theta = \frac{1}{x}\right)$$

H.W. Problems :-

1- Find : $\lim_{n\to -2^-}(x+3)^{\frac{|x+2|}{x+2}}$

2- What value should be assigned to (a) to make the function :

$$f(x)=\begin{cases} x^2-1 & , \quad x<3 \\ 2ax & , \quad x\geq 3 \end{cases}$$

Continuous at x = 3 ?

3- At what point is the function f(x) continuous ?

$$f(x)=\begin{cases} 1 & , \quad x<0 \\ \sqrt{1-x} & , \quad 0\leq x\leq 1 \\ x-1 & , \quad x>1 \end{cases}$$

4- $\lim_{n\to\infty}(5x^3-2x^2)$

5- $\lim_{n\to\infty}\frac{x^{\frac{3}{2}}+5}{\sqrt{x^3+4}}$

6- Let :

$$f(x)=\begin{cases} 1-x^2 & , \quad x\neq 1 \\ 2 & , \quad x=1 \end{cases}$$

a) Find $\lim_{n\to 1^+} f(x)$ and $\lim_{n\to 1^-} f(x)$.

b) Does $\lim_{n\to 1} f(x)$ exist ? why ?

c) Graph f(x) .