4.4 Overall Mass Balance and Continuity Equation

In fluid dynamics, fluids are in motion. Generally, they are moved from place to place by means of mechanical devices such as pumps or blowers, by gravity head, or by pressure, and flow through systems of piping and/or process equipment.

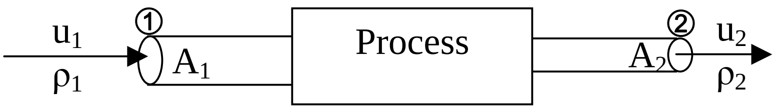
The first step in the solution of flow problems is generally to apply the principles of the conservation of mass to the whole system or any part of the system.

INPUT – OUTPUT = ACCUMULATION

At steady state, the rate of accumulation is zero

∴ INPUT = OUTPUT

In the following Figure a simple flow system is shown where fluid enters section ① with an average velocity (u_1) and density (ρ_1) through the cross-sectional area (A_1). The fluid leaves section ② with an average velocity (u_2) and density (ρ_1) through the cross-sectional area (A_2).



Thus,

At steady state $\dot{m}_1 = \dot{m}2$

$$Q_1\ \rho_1 = Q_2\ \rho_2$$

$$u_1\ A_1\ \rho_1 = u_2\ A_2\ \rho_2$$

For incompressible fluids at the same temperature [$\rho_1 = \rho_2$]

$$\therefore\ u_1\ A_1 = u_2\ A_2$$

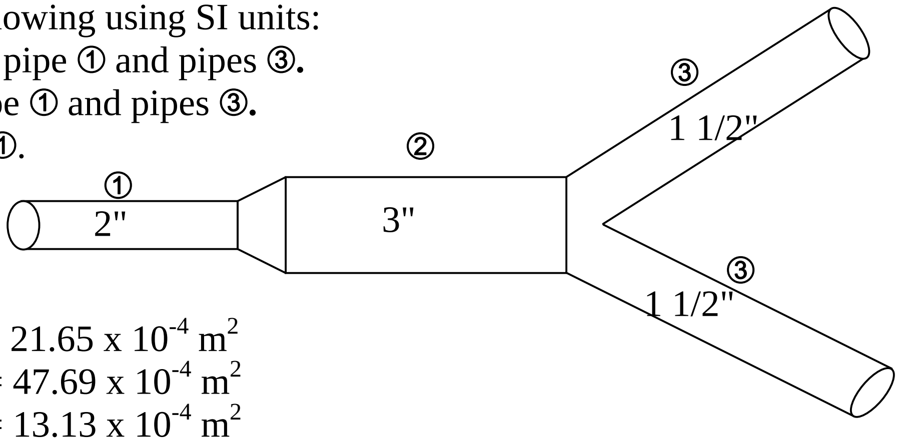
Example -4.2-

A petroleum crude oil having a density of 892 kg/m^3 is flowing, through the piping arrangement shown in the below Figure, at total rate of 1.388 x 10^{-3} m^3/s entering pipe ①. The flow divides equally in each of pipes ③. The steel pipes are schedule 40 pipe. **Table{{{{}}}.** Calculate the following using SI units:

a- The total mass flow rate in pipe ① and pipes ③**.**

b- The average velocity in pipe ① and pipes ③**.**

c- The mass velocity in pipe ①.



Solution:

Pipe ① I.D. = 0.0525 m, A1 = 21.65 x 10^{-4} m^2

Pipe ② I.D. = 0.07792 m, A_1 = 47.69 x 10^{-4} m^2

Pipe ③ I.D. = 0.04089 m, A_1 = 13.13 x 10^{-4} m^2

a- the total mass flow rate is the same through pipes ① and ② and is

$\dot{m}_1 = Q_1\,\rho$ = 1.388 x 10^{-3} m^3/s (892 kg/m^3) = 1.238 kg/s

Since the flow divides equally in each pipes ③'

$\Rightarrow\ \dot{m}_3 = \dot{m}_1/2 = 1.238/2 =$ 0.619 kg/s

b- $\dot{m}_1 = Q_1\,\rho = u_1 A_1\,\rho\ \Rightarrow\ u_1 = \dfrac{\dot{m}_1}{A_1\,\rho} = \dfrac{1.238\ kg/s}{(21.65\times10^{-4}\,m^2)(892\ kg/m^3)}$ = 0.641 m/s

$u_3 = \dfrac{\dot{m}_3}{A_3\,\rho} = \dfrac{0.619\ kg/s}{(13.13\times10^{-4}\,m^2)(892\ kg/m^3)}$ = 0.528 m/s

d- $G_1 = u_1\,\rho_1$ = 0.641 m/s (892 kg/m^3) = 572 kg/m^2.s

e- or $G_1 = \dfrac{\dot{m}_1}{A_1} = \dfrac{1.238\ kg/s}{21.65\times10^{-4}\,m^2}$ = 572 kg/m^2.s

4.5 Energy Relationships and Bernoulli's Equation

The total energy of a fluid in motion consists of the following components: -

Internal Energy (U)

This is the energy associated with the physical state of fluid, i.e. the energy of atoms and molecules resulting from their motion and configuration. Internal energy is a function of temperature. It can be written as (U) energy per unit mass of fluid.

Potential Energy (PE)

This is the energy that a fluid has because of its position in the earth's field of gravity. The work required to raise a unit mass of fluid to a height (z) above a datum line is (zg), where (g) is gravitational acceleration. This work is equal to the potential energy per unit mass of fluid above the datum line.

Kinetic Energy (KE)

This is the energy associated with the physical state of fluid motion. The kinetic energy of unit mass of the fluid is (u^2/2), where (u) is the linear velocity of the fluid relative to some fixed body.

Pressure Energy (Prss.E)

This is the energy or work required to introduce the fluid into the system without a change in volume. If (P) is the pressure and (V) is the volume of a mass (m) of fluid, then ($PV/m \equiv P\upsilon$) is the pressure energy per unit mass of fluid. The ratio (V/m) is the fluid density (ρ).

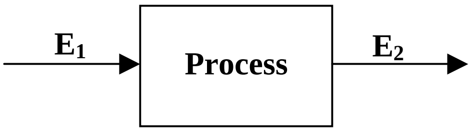
The total energy (E) per unit mass of fluid is given by the equation: -

$$\boxed{E = U + zg + P/\rho + u^2/2}$$

where, each term has the dimension of force times distance per unit mass. In calculation, each term in the equation must be expressed in the same units, such as J/kg, Btu/lb or lb_f.ft/lb. i.e. $(MLT^{-2})(L)(M^{-1}) = [L^2T^{-2}] \equiv \{m^2/s^2, ft^2/s^2\}$.

A flowing fluid is required to do work in order to overcome viscous frictional forces that resist the flow.

The principle of the conservation of energy will be applied to a process of input and output streams for ideal fluid of constant density and without any pump present and no change in temperature.



$$E_1 = E_2$$

$$U_1 + z_1 g + P_1/\rho + u_1^2/2 = U_2 + z_2 g + P_2/\rho + u_2^2/2$$

$U_1 = U_2$ (no change in temperature)

$$P_1/\rho + u_1^2/2 + z_1 g = P_2/\rho + u_2^2/2 + z_2 g$$

$$\Rightarrow P/\rho + u^2/2 + z g = \text{constant}$$

$$\Rightarrow \Delta P/\rho + \Delta u^2/2 + \Delta z g = 0$$ --------- Bernoulli's equation