1. Fluid Flow in Pipes

We will be looking here at the flow of real fluid in pipes – *real* meaning a fluid that possesses viscosity hence looses energy due to friction as fluid particles interact with one another and the pipe wall.

Recall from Level 1 that the shear stress induced in a fluid flowing near a boundary is given by Newton's law of viscosity:

$$\tau \propto \frac{du}{dy}$$

This tells us that the shear stress, τ, in a fluid is proportional to the velocity gradient - the rate of change of velocity across the fluid path. For a "Newtonian" fluid we can write:

$$\tau = \mu \frac{du}{dy}$$

where the constant of proportionality, μ, is known as the coefficient of viscosity (or simply viscosity).

Recall also that flow can be classified into one of two types, **laminar** or **turbulent** flow (with a small transitional region between these two). The non-dimensional number, the Reynolds number, Re, is used to determine which type of flow occurs:

$$\mathrm{Re} = \frac{\rho u d}{\mu}$$

For a pipe

Laminar flow:	$\mathrm{Re} < 2000$
Transitional flow:	$2000 < \mathrm{Re} < 4000$
Turbulent flow:	$\mathrm{Re} > 4000$

It is important to determine the flow type as this governs how the amount of energy lost to friction relates to the velocity of the flow. And hence how much energy must be used to move the fluid.

1.1 Pressure loss due to friction in a pipeline.

Consider a cylindrical element of incompressible fluid flowing in the pipe, as shown

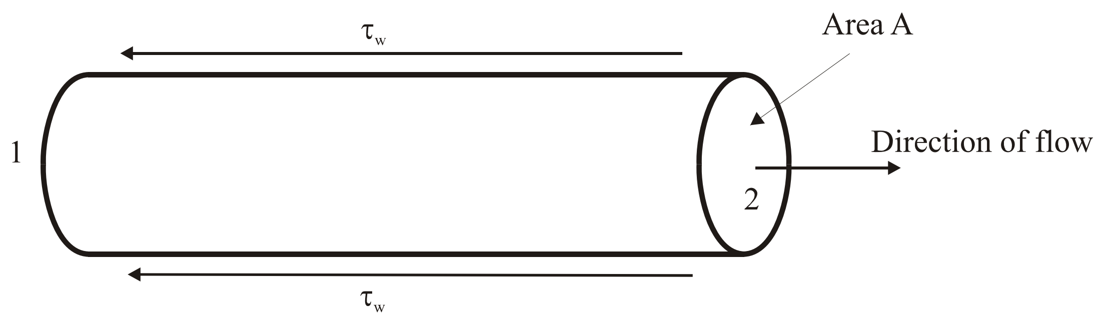

Figure 1: Element of fluid in a pipe

The pressure at the upstream end, 1, is *p*, and at the downstream end, 2, the pressure has fallen by Δp to (p-Δp).

The driving force due to pressure (F = Pressure x Area) can then be written

driving force = Pressure force at 1 - pressure force at 2

$$pA - (p - \Delta p)A = \Delta p\, A = \Delta p \frac{\pi d^2}{4}$$

The retarding force is that due to the shear stress by the walls

$$= \text{shear stress} \times \text{area over which it acts}$$
$$= \tau_w \times \text{area of pipe wall}$$
$$= \tau_w \pi d L$$

As the flow is in equilibrium,

driving force = retarding force

$$\Delta p \frac{\pi d^2}{4} = \tau_w \pi d L$$

$$\Delta p = \frac{\tau_w 4L}{d}$$

Equation 1

Giving an expression for pressure loss in a pipe in terms of the pipe diameter and the shear stress at the wall on the pipe.

The shear stress will vary with velocity of flow and hence with Re. Many experiments have been done with various fluids measuring the pressure loss at various Reynolds numbers. These results plotted to show a graph of the relationship between pressure loss and Re look similar to the figure below:

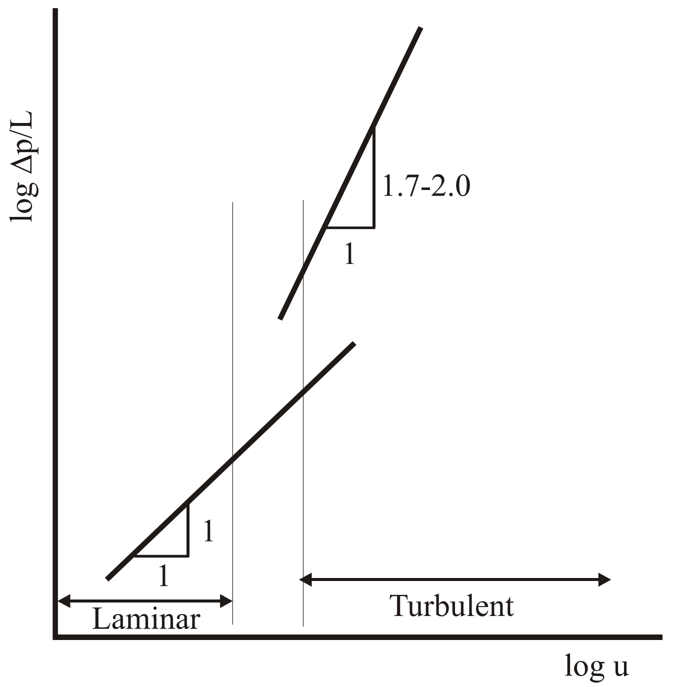

Figure 2: Relationship between velocity and pressure loss in pipes