This graph shows that the relationship between pressure loss and Re can be expressed as

laminar $\Delta p \propto u$

turbulent $\Delta p \propto u^a$

where $1.7 < a < 2.0$

As these are empirical relationships, they help in determining the pressure loss but not in finding the magnitude of the shear stress at the wall τ_w on a particular fluid. If we knew τ_w we could then use it to give a general equation to predict the pressure loss.

1.2 Pressure loss during laminar flow in a pipe

In general the shear stress τ_w. is almost impossible to measure. But for laminar flow it is possible to calculate a theoretical value for a given velocity, fluid and pipe dimension. (As this was covered in he Level 1 module, only the result is presented here.) The pressure loss in a pipe with laminar flow is given by the Hagen-Poiseuille equation:

$$\Delta p = \frac{32\mu L u}{d^2}$$

or in terms of head

$$h_f = \frac{32\mu L u}{\rho g d^2}$$

Equation 2

Where $\boldsymbol{h_f}$ is known as the ***head-loss due to friction***

(Remember the velocity, u, is means velocity – and is sometimes written $\bar{u}$.)

1.3 Pressure loss during turbulent flow in a pipe

In this derivation we will consider a general bounded flow - fluid flowing in a channel - we will then apply this to pipe flow. In general it is most common in engineering to have Re > 2000 i.e. turbulent flow – in both closed (pipes and ducts) and open (rivers and channels). However analytical expressions are not available so empirical relationships are required (those derived from experimental measurements).

Consider the element of fluid, shown in figure 3 below, flowing in a channel, it has length L and with wetted perimeter P. The flow is steady and uniform so that acceleration is zero and the flow area at sections 1 and 2 is equal to A.

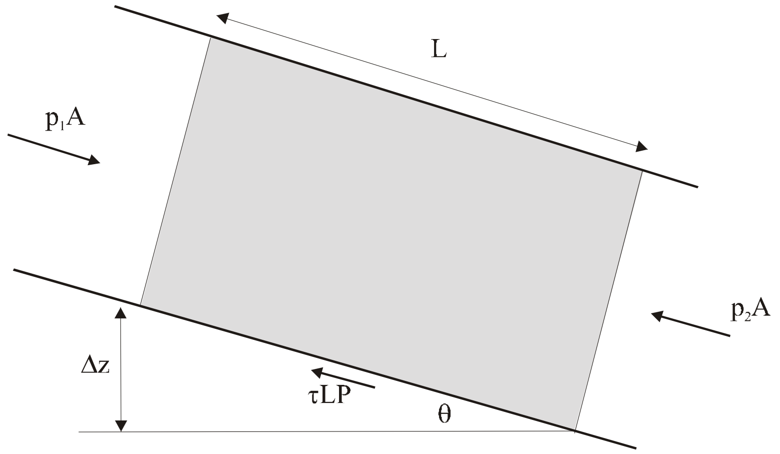

Figure 3: Element of fluid in a channel flowing with uniform flow

$$p_1 A - p_2 A - \tau_w LP + W \sin\theta = 0$$

writing the weight term as ρgAL and sin θ = –Δz/L gives

$$A(p_1 - p_2) - \tau_w LP - \rho gA\Delta z = 0$$

this can be rearranged to give

$$\frac{[(p_1 - p_2) - \rho g\Delta z]}{L} - \tau_o \frac{P}{A} = 0$$

where the first term represents the piezometric head loss of the length L or (writing piezometric head p^*)

$$\tau_o = m\frac{dp^*}{dx}$$

Equation 3

where $m = A/P$ is known as the hydraulic mean depth

Writing piezometric head loss as $p^* = \rho gh_f$, then shear stress per unit length is expressed as

$$\tau_o = m\frac{dp^*}{dx} = m\frac{\rho gh_f}{L}$$

So we now have a relationship of shear stress at the wall to the rate of change in piezometric pressure. To make use of this equation an empirical factor must be introduced. This is usually in the form of a **friction factor *f***, and written

$$\tau_o = f\frac{\rho u^2}{2}$$

where u is the mean flow velocity.

Hence

$$\frac{dp^*}{dx} = f\frac{\rho u^2}{2m} = \frac{\rho gh_f}{L}$$

So, for a general bounded flow, head loss due to friction can be written

$$h_f = \frac{fLu^2}{2m}$$

Equation 4

More specifically, for a circular pipe, $m = A/P = \pi d^2/4\pi d = d/4$ giving

$$h_f = \frac{4fLu^2}{2gd}$$

Equation 5

This is known as the **Darcy-Weisbach** equation for head loss in circular pipes

(Often referred to as the Darcy equation)

This equation is equivalent to the Hagen-Poiseuille equation for laminar flow with the exception of the empirical friction factor f introduced.

It is sometimes useful to write the Darcy equation in terms of discharge Q, (using $Q = Au$)