$$u = \frac{4Q}{\pi d^2}$$

$$h_f = \frac{64\, fLQ^2}{2g\pi^2 d^5} = \frac{fLQ^2}{3.03 d^5}$$

Equation 6

Or with a 1% error

$$h_f = \frac{fLQ^2}{3d^5}$$

Equation 7

NOTE On Friction Factor Value

The f value shown above is different to that used in American practice. Their relationship is

$$f_{American} = 4f$$

Sometimes the f is replaced by the Greek letter λ. where

$$\lambda = f_{American} = 4f$$

Consequently great care must be taken when choosing the value of f with attention taken to the source of that value.

1.4 Choice of friction factor f

The value of f must be chosen with care or else the head loss will not be correct. Assessment of the physics governing the value of friction in a fluid has led to the following relationships

1. $h_f \propto L$
2. $h_f \propto v^2$
3. $h_f \propto 1/d$
4. h_f depends on surface roughness of pipes
5. h_f depends on fluid density and viscosity
6. h_f is independent of pressure

Consequently f cannot be a constant if it is to give correct head loss values from the Darcy equation. An expression that gives f based on fluid properties and the flow conditions is required.

1.4.1 The value of *f* for Laminar flow

As mentioned above the equation derived for head loss in turbulent flow is equivalent to that derived for laminar flow – the only difference being the empirical *f*. Equation the two equations for head loss allows us to derive an expression of *f* that allows the Darcy equation to be applied to laminar flow.

Equating the Hagen-Poiseuille and Darcy-Weisbach equations gives:

$$\frac{32\mu Lu}{\rho g d^2} = \frac{4fLu^2}{2gd}$$

$$f = \frac{16\mu}{\rho v d}$$

$$f = \frac{16}{\mathrm{Re}}$$

Equation 8

1.4.2 Blasius equation for f

Blasius, in 1913, was the first to give an accurate empirical expression for *f* for turbulent flow in smooth pipes, that is:

$$f = \frac{0.079}{\mathrm{Re}^{0.25}}$$

Equation 9

This expression is fairly accurate, giving head losses +/- 5% of actual values for Re up to 100000.

1.4.3 Nikuradse

Nikuradse made a great contribution to the theory of pipe flow by differentiating between rough and smooth pipes. A rough pipe is one where the mean height of roughness is greater than the thickness of the laminar sub-layer. Nikuradse artificially roughened pipe by coating them with sand. He defined a *relative roughness* value k_s/d (mean height of roughness over pipe diameter) and produced graphs of *f* against Re for a range of relative roughness 1/30 to 1/1014.

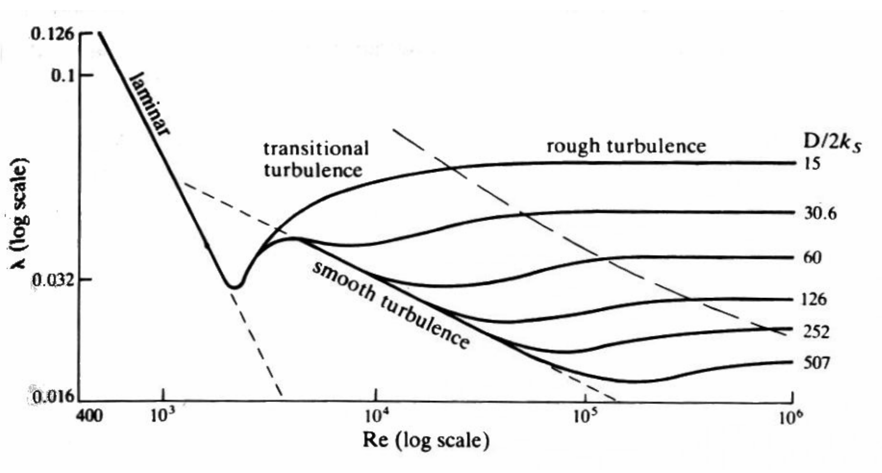

Figure 4: Regions on plot of Nikurades's data