A number of distinct regions can be identified on the diagram.

The regions which can be identified are:

1. Laminar flow (*f* = 16/Re)
2. Transition from laminar to turbulent
   An unstable region between Re = 2000 and 4000. Pipe flow normally lies outside this region
3. Smooth turbulent
   The limiting line of turbulent flow. All values of relative roughness tend toward this as Re decreases.
4. Transitional turbulent
   The region which *f* varies with both Re and relative roughness. Most pipes lie in this region.
5. Rough turbulent. *f* remains constant for a given relative roughness. It is independent of Re.

1.4.4 Colebrook-White equation for f

Colebrook and White did a large number of experiments on commercial pipes and they also brought together some important theoretical work by von Karman and Prandtl. This work resulted in an equation attributed to them as the Colebrook-White equation:

$$\frac{1}{\sqrt{f}} = -4\log_{10}\left(\frac{k_s}{3.71d} + \frac{1.26}{\mathrm{Re}\sqrt{f}}\right)$$

Equation 10

It is applicable to the whole of the turbulent region for commercial pipes and uses an effective roughness value (k_s) obtained experimentally for all commercial pipes.

Note a particular difficulty with this equation. *f* appears on both sides in a square root term and so cannot be calculated easily. Trial and error methods must be used to get *f* once k_s¸Re and d are known. (In the 1940s when calculations were done by slide rule this was a time consuming task.) Nowadays it is relatively trivial to solve the equation on a programmable calculator or spreadsheet.

Moody made a useful contribution to help, he plotted *f* against *Re* for commercial pipes – see the figure below. This figure has become known as the Moody Diagram. [Note that this figure uses λ (= 4*f*) for friction factor rather than *f*. The shape of the diagram will not change if *f* were used instead.]

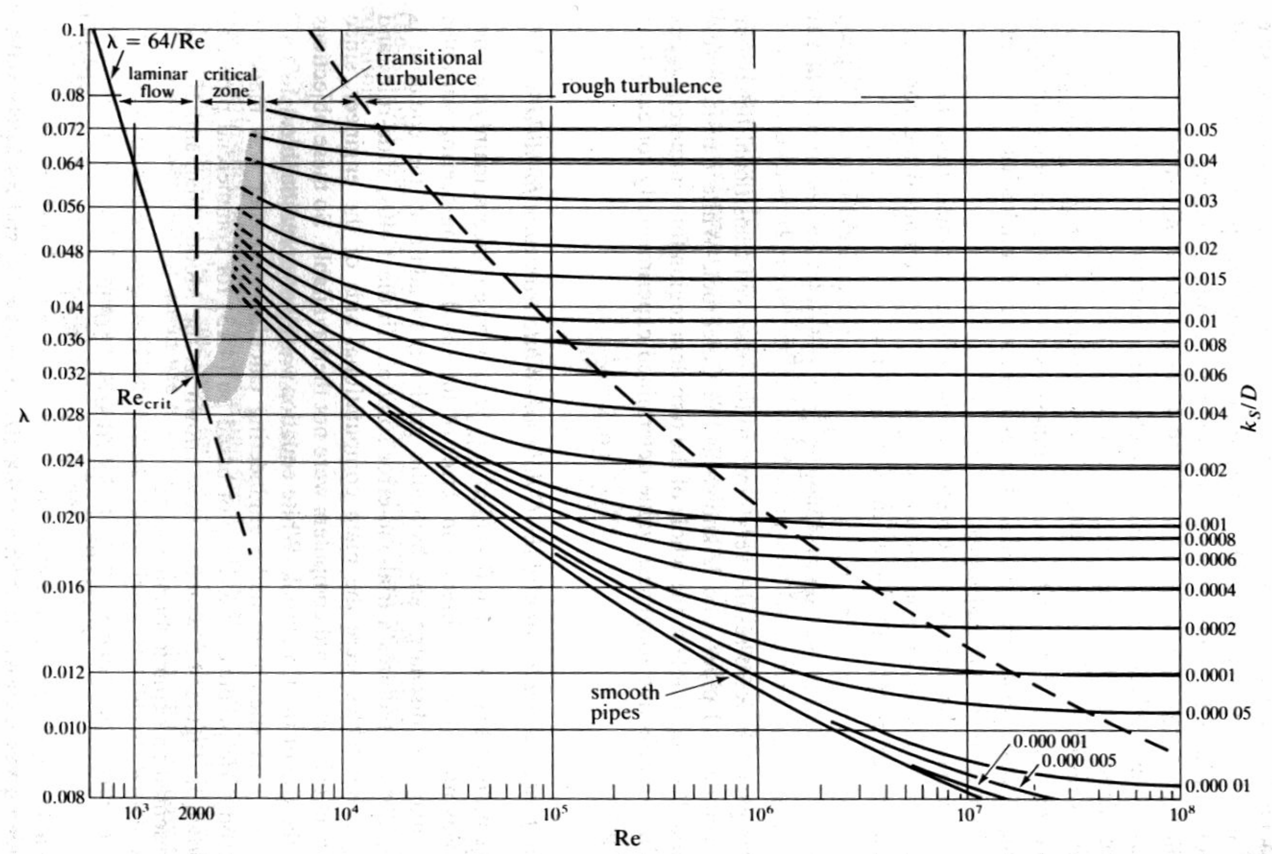

Figure 5: Moody Diagram.

He also developed an equation based on the Colebrook-White equation that made it simpler to calculate *f*:

$$f = 0.001375\left[1+\left(\frac{200k_s}{d}+\frac{10^6}{\mathrm{Re}}\right)^{1/3}\right]$$

Equation 11

This equation of Moody gives *f* correct to +/- 5% for $4 \times 10^3 < \mathrm{Re} < 1 \times 10^7$ and for $k_s/d < 0.01$.

Barr presented an alternative explicit equation for *f* in 1975

$$\frac{1}{\sqrt{f}} = -4\log_{10}\left[\frac{k_s}{3.71d}+\frac{5.1286}{\mathrm{Re}^{0.89}}\right]$$

Equation 12

or

$$f = 1\Big/\left[-4\log_{10}\left(\frac{k_s}{3.71d}+\frac{5.1286}{\mathrm{Re}^{0.89}}\right)\right]^2$$

Equation 13

Here the last term of the Colebrook-White equation has been replaced with $5.1286/\mathrm{Re}^{0.89}$ which provides more accurate results for $\mathrm{Re} > 10^5$.

The problem with these formulas still remains that these contain a dependence on k_s. What value of k_s should be used for any particular pipe? Fortunately pipe manufactures provide values and typical values can often be taken similar to those in table 1 below.