1.6 Pipeline Analysis

To analyses the flow in a pipe line we will use Bernoulli's equation. The Bernoulli equation was introduced in the Level 1 module, and as a reminder it is presented again here.

Bernoulli's equation is a statement of conservation of energy along a streamline, by this principle the total *energy* in the system does not change, Thus the total *head* does not change. So the Bernoulli equation can be written

$$\frac{p}{\rho g} + \frac{u^2}{2g} + z = H = \text{constant}$$

or

$$\begin{matrix} \text{Pressure} & & \text{Kinetic} & & \text{Potential} & & \text{Total} \\ \text{energy per} & + & \text{energy per} & + & \text{energy per} & = & \text{energy per} \\ \text{unit weight} & & \text{unit weight} & & \text{unit weight} & & \text{unit weight} \end{matrix}$$

As all of these elements of the equation have units of length, they are often referred to as the following:

pressure head = $\dfrac{p}{\rho g}$

velocity head = $\dfrac{u^2}{2g}$

potential head = z

total head = H

In this form Bernoulli's equation has some restrictions in its applicability, they are:

- Flow is steady;
- Density is constant (i.e. fluid is incompressible);
- Friction losses are negligible.
- The equation relates the states at two points along a single streamline.

1.7 Pressure Head, Velocity Head, Potential Head and Total Head in a Pipeline.

By looking at the example of the reservoir with which feeds a pipe we will see how these different *heads* relate to each other.

Consider the reservoir below feeding a pipe that changes diameter and rises (in reality it may have to pass over a hill) before falling to its final level.

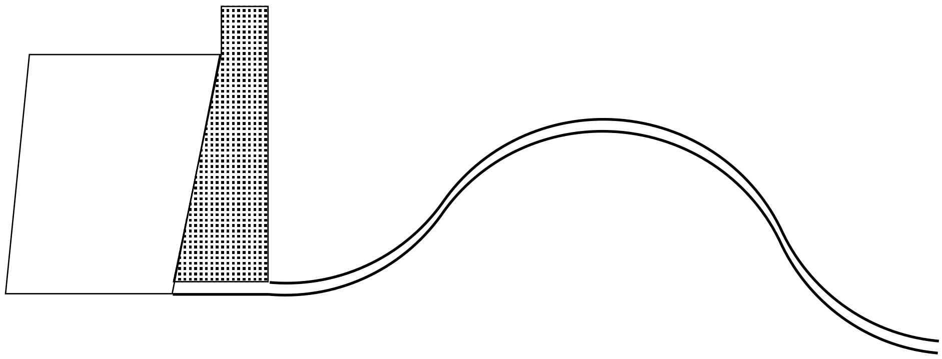

Figure 10: Reservoir feeding a pipe

To analyses the flow in the pipe we apply the Bernoulli equation along a streamline from point 1 on the surface of the reservoir to point 2 at the outlet nozzle of the pipe. And we know that the *total energy per unit weight* or the *total head* does not change - it is **constant** - along a streamline. But what is this value of this constant? We have the Bernoulli equation

$$\frac{p_1}{\rho g}+\frac{u_1^2}{2g}+z_1=H=\frac{p_2}{\rho g}+\frac{u_2^2}{2g}+z_2$$

We can calculate the total head, *H*, at the reservoir, $p_1 = 0$ as this is atmospheric and atmospheric gauge pressure is zero, the surface is moving very slowly compared to that in the pipe so $u_1 = 0$, so all we are left with is $total\,head = H = z_1$ the elevation of the reservoir.

A useful method of analysing the flow is to show the pressures graphically on the same diagram as the pipe and reservoir. In the figure above the *total head* line is shown. If we attached piezometers at points along the pipe, what would be their levels when the pipe nozzle was closed? (Piezometers, as you will remember, are simply open ended vertical tubes filled with the same liquid whose pressure they are measuring).

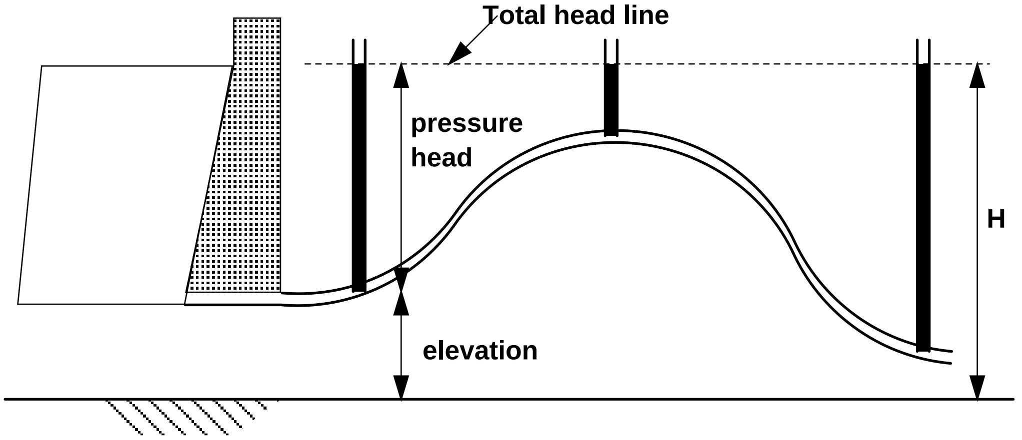

Figure 11: Piezometer levels with zero velocity

As you can see in the above figure, with zero velocity all of the levels in the piezometers are equal and the same as the total head line. At each point on the line, when *u = 0*