

The pipe diameter is 100mm and has length 15m and feeds directly into the atmosphere at point C 4m below the surface of the reservoir (i.e. $z_a - z_c = 4.0\text{m}$). The highest point on the pipe is a B which is 1.5m above the surface of the reservoir (i.e. $z_b - z_a = 1.5\text{m}$) and 5 m along the pipe measured from the reservoir. Assume the entrance and exit to the pipe to be sharp and the value of friction factor f to be 0.08. Calculate a) velocity of water leaving the pipe at point C, b) pressure in the pipe at point B.

a)

We use the Bernoulli equation with appropriate losses from point A to C
and for entry loss $k_L = 0.5$ and exit loss $k_L = 1.0$.

For the local losses from Table 2 for a sharp entry $k_L = 0.5$ and for the sharp exit as it opens in to the atmosphere with no contraction there are no losses, so

$$h_L = 0.5 \frac{u^2}{2g}$$

Friction losses are given by the Darcy equation

$$h_f = \frac{4fLu^2}{2gd}$$

Pressure at A and C are both atmospheric, u_A is very small so can be set to zero, giving

$$z_A = \frac{u^2}{2g} + z_C + \frac{4fLu^2}{2gd} + 0.5 \frac{u^2}{2g}$$

$$z_A - z_C = \frac{u^2}{2g} \left(1 + 0.5 + \frac{4fL}{d} \right)$$

Substitute in the numbers from the question

$$4 = \frac{u^2}{2 \times 9.81} \left(1.5 + \frac{4 \times 0.08 \times 15}{0.1} \right)$$

$$u = 1.26 \text{ m/s}$$

b)

To find the pressure at B apply Bernoulli from point A to B using the velocity calculated above. The length of the pipe is $L_1 = 5\text{m}$:

$$z_A = \frac{p_B}{\rho g} + \frac{u^2}{2g} + z_B + \frac{4fL_1 u^2}{2gd} + 0.5 \frac{u^2}{2g}$$

$$z_A - z_B = \frac{p_B}{\rho g} + \frac{u^2}{2g} \left(1 + 0.5 + \frac{4fL_1}{d} \right)$$

$$-1.5 = \frac{p_B}{1000 \times 9.81} + \frac{1.26^2}{2 \times 9.81} \left(1.5 + \frac{4 \times 0.08 \times 5.0}{0.1} \right)$$

$$p_B = -28.58 \times 10^3 \text{ N/m}^2$$

That is 28.58 kN/m² **below** atmospheric.

1.10 Pipes in series

When pipes of different diameters are connected end to end to form a pipe line, they are said to be in series. The total loss of energy (or head) will be the sum of the losses in each pipe plus local losses at connections.

1.10.1 Pipes in Series Example

Consider the two reservoirs shown in figure 16, connected by a single pipe that changes diameter over its length. The surfaces of the two reservoirs have a difference in level of 9m. The pipe has a diameter of 200mm for the first 15m (from A to C) then a diameter of 250mm for the remaining 45m (from C to B).

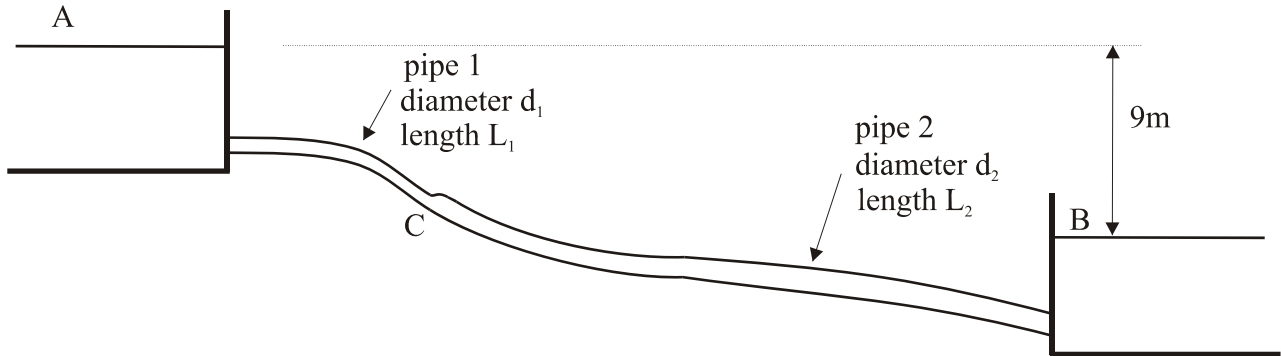


Figure 16:

For the entrance use $k_L = 0.5$ and the exit $k_L = 1.0$. The join at C is sudden. For both pipes use $f = 0.01$.

Total head loss for the system H = height difference of reservoirs

h_{f1} = head loss for 200mm diameter section of pipe

h_{f2} = head loss for 250mm diameter section of pipe

$h_{L \text{ entry}}$ = head loss at entry point

$h_{L \text{ join}}$ = head loss at join of the two pipes

$h_{L \text{ exit}}$ = head loss at exit point

So

$$H = h_{f1} + h_{f2} + h_{L \text{ entry}} + h_{L \text{ join}} + h_{L \text{ exit}} = 9\text{m}$$

All losses are, in terms of Q :

$$h_{f1} = \frac{fL_1Q^2}{3d_1^5}$$

$$h_{f2} = \frac{fL_2Q^2}{3d_2^5}$$

$$h_{L \text{ entry}} = 0.5 \frac{u_1^2}{2g} = 0.5 \frac{1}{2g} \left(\frac{4Q}{\pi d_1^2} \right)^2 = 0.5 \times 0.0826 \frac{Q^2}{d_1^4} = 0.0413 \frac{Q^2}{d_1^4}$$

$$h_{L \text{ exit}} = 1.0 \frac{u_2^2}{2g} = 1.0 \times 0.0826 \frac{Q^2}{d_2^4} = 0.0826 \frac{Q^2}{d_2^4}$$

$$h_{L \text{ join}} = \frac{(u_1 - u_2)^2}{2g} = \left(\frac{4Q}{\pi} \right)^2 \frac{\left(\frac{1}{d_1^2} - \frac{1}{d_2^2} \right)^2}{2g} = 0.0826 Q^2 \left(\frac{1}{d_1^2} - \frac{1}{d_2^2} \right)^2$$

Substitute these into

$$h_{f1} + h_{f2} + h_{L \text{ entry}} + h_{L \text{ join}} + h_{L \text{ exit}} = 9$$

and solve for Q , to give $Q = 0.158 \text{ m}^3/\text{s}$