1.11 Pipes in parallel

When two or more pipes in parallel connect two reservoirs, as shown in Figure 17, for example, then the fluid may flow down any of the available pipes at possible different rates. But the head difference over each pipe will always be the same. The total volume flow rate will be the sum of the flow in each pipe.

The analysis can be carried out by simply treating each pipe individually and summing flow rates at the end.

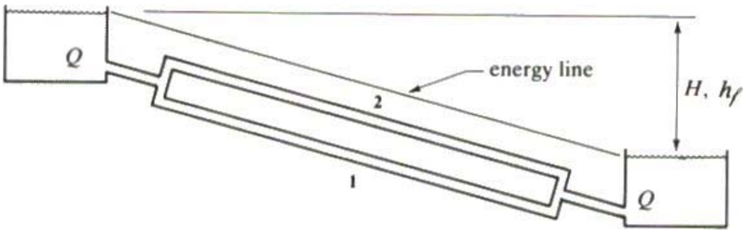

Figure 17: Pipes in Parallel

1.11.1 Pipes in Parallel Example

Two pipes connect two reservoirs (A and B) which have a height difference of 10m. Pipe 1 has diameter 50mm and length 100m. Pipe 2 has diameter 100mm and length 100m. Both have entry loss k_L = 0.5 and exit loss k_L=1.0 and Darcy *f* of 0.008.
Calculate:

a) rate of flow for each pipe
b) the diameter D of a pipe 100m long that could replace the two pipes and provide the same flow.

a)
Apply Bernoulli to each pipe separately. For pipe 1:

$$\frac{p_A}{\rho g}+\frac{u_A^2}{2g}+z_A=\frac{p_B}{\rho g}+\frac{u_B^2}{2g}+z_B+0.5\frac{u_1^2}{2g}+\frac{4flu_1^2}{2gd_1}+1.0\frac{u_1^2}{2g}$$

p_A and p_B are atmospheric, and as the reservoir surface move s slowly u_A and u_B are negligible, so

$$z_A-z_B=\left(0.5+\frac{4fl}{d_1}+1.0\right)\frac{u_1^2}{2g}$$

$$10=\left(1.0+\frac{4\times 0.008\times 100}{0.05}\right)\frac{u_1^2}{2\times 9.81}$$

$$u_1=1.731m/s$$

And flow rate is given by

$$Q_1=u_1\frac{\pi d_1^2}{4}=0.0034m^3/s$$

For pipe 2:

$$\frac{p_A}{\rho g}+\frac{u_A^2}{2g}+z_A=\frac{p_B}{\rho g}+\frac{u_B^2}{2g}+z_B+0.5\frac{u_2^2}{2g}+\frac{4flu_2^2}{2gd_2}+1.0\frac{u_2^2}{2g}$$

Again p_A and p_B are atmospheric, and as the reservoir surface move s slowly u_A and u_B are negligible, so

$$z_A - z_B = \left(0.5 + \frac{4fl}{d_2} + 1.0\right)\frac{u_2^2}{2g}$$

$$10 = \left(1.0 + \frac{4\times 0.008\times 100}{0.1}\right)\frac{u_2^2}{2\times 9.81}$$

$$u_2 = 2.42\,m/s$$

And flow rate is given by

$$Q_2 = u_2\frac{\pi d_2^2}{4} = 0.0190\,m^3/s$$

b) Replacing the pipe, we need $Q = Q_1 + Q_2 = 0.0034 + 0.0190 = 0.0224\ m^3/s$

For this pipe, diameter D, velocity u , and making the same assumptions about entry/exit losses, we have:

$$\frac{p_A}{\rho g} + \frac{u_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{u_B^2}{2g} + z_B + 0.5\frac{u^2}{2g} + \frac{4flu^2}{2gD} + 1.0\frac{u^2}{2g}$$

$$z_A - z_B = \left(0.5 + \frac{4fl}{D} + 1.0\right)\frac{u^2}{2g}$$

$$10 = \left(1.0 + \frac{4\times 0.008\times 100}{D}\right)\frac{u^2}{2\times 9.81}$$

$$196.2 = \left(1.0 + \frac{3.2}{D}\right)u^2$$

The velocity can be obtained from Q i.e.

$$Q = Au = \frac{\pi D^2}{4}u$$

$$u = \frac{4Q}{\pi D^2} = \frac{0.02852}{D^2}$$

So

$$196.2 = \left(1.0 + \frac{3.2}{D}\right)\left(\frac{0.02852}{D^2}\right)^2$$

$$0 = 241212D^5 - 1.5D - 3.2$$

which must be solved iteratively

An approximate answer can be obtained by dropping the second term:

$$0 = 241212D^5 - 3.2$$

$$D = \sqrt[5]{3.2/241212}$$

$$D = 0.1058m$$

Writing the function

$$f(D) = 241212D^5 - 1.5D - 3.2$$

$$f(0.1058) = -0.161$$

So increase D slightly, try 0.107m

$$f(0.107) = 0.022$$

.i.e. the solution is between 0.107m and 0.1058m but 0.107 if sufficiently accurate