The algebraic solution is rather tedious so a trial and error method is usually recommended. For example this procedure usually converges to a solution quickly:

1. estimate a value of the head at the junction, $h_D$
2. substitute this into the first three equations to get an estimate for Q for each pipe.
3. check to see if continuity is (or is not) satisfied from the fourth equation
4. if the flow into the junction is too high choose a larger $h_D$ and vice versa.
5. return to step 2

If the direction of the flow in pipe 2 was wrongly assumed then no solution will be found. If you have made this mistake then switch the direction to obtain these four equations

$$z_a - h_D = h_{f1}$$

$$z_b - h_D = h_{f2}$$

$$h_D - z_c = h_{f3}$$

$$Q_1 + Q_2 = Q_3$$

Looking at these two sets of equations we can see that they are identical if $h_D = z_b$. This suggests that a good starting value for the iteration is $z_b$ then the direction of flow will become clear at the first iteration.

### 1.12.1 Example of Branched Pipe – The Three Reservoir Problem

Water flows from reservoir A through pipe 1, diameter $d_1$ = 120mm, length $L_1$=120m, to junction D from which the two pipes leave, pipe 2, diameter $d_2$=75mm, length $L_2$=60m goes to reservoir B, and pipe 3, diameter $d_3$=60mm, length $L_3$=40m goes to reservoir C. Reservoir B is 16m below reservoir A, and reservoir C is 24m below reservoir A. All pipes have $f = 0.01$. (Ignore and entry and exit losses.)

We know the flow is from A to D and from D to C but are never quite sure which way the flow is along the other pipe – either D to B or B to D. We first must assume one direction. If that is not correct there will not be a sensible solution. To keep the notation from above we can write $z_a = 24$, $z_b = 16$ and $z_c = 0$.

For flow A to D

$$z_a - h_D = h_{f1}$$

$$24 - h_D = \frac{f_1 L_1 Q_1^2}{3 d_1^5} = 16075\, Q_1^2$$

Assume flow is D to B

$$h_D - z_b = h_{f2}$$

$$h_D - 8 = \frac{f_2 L_2 Q_2^2}{3 d_2^5} = 84280\, Q_2^2$$

For flow is D to C

$$h_D - z_c = h_{f3}$$

$$h_D - 0 = \frac{f_3 L_3 Q_3^2}{3 d_3^5} = 171468\, Q_3^2$$

The final equation is continuity, which for this chosen direction D to B is

$$Q_1 = Q_2 + Q_3$$

Now it is a matter of systematically questing values of $h_D$ until continuity is satisfied. This is best done in a table. And it is usually best to initially guess $h_D = z_a$ then reduce its value (until the error in continuity is small):

| hj | Q1 | Q2 | Q3 | Q1=Q2+Q3 | err |
|---|---|---|---|---|---|
| 24.00 | 0.00000 | 0.01378 | 0.01183 | 0.02561 | 0.02561 |
| 20.00 | 0.01577 | 0.01193 | 0.01080 | 0.02273 | 0.00696 |
| 17.00 | 0.02087 | 0.01033 | 0.00996 | 0.02029 | -0.00058 |
| 17.10 | 0.02072 | 0.01039 | 0.00999 | 0.02038 | -0.00034 |
| 17.20 | 0.02057 | 0.01045 | 0.01002 | 0.02046 | -0.00010 |
| 17.30 | 0.02042 | 0.01050 | 0.01004 | 0.02055 | 0.00013 |
| 17.25 | 0.02049 | 0.01048 | 0.01003 | 0.02051 | 0.00001 |
| 17.24 | 0.02051 | 0.01047 | 0.01003 | 0.02050 | -0.00001 |

So the solution is that the head at the junction is 17.24 m, which gives $Q_1 = 0.0205 m^3/s$, $Q_1 = 0.01047 m^3/s$ and $Q_1 = 0.01003 m^3/s$.

Had we guessed that the flow was from B to D, the second equation would have been

$$z_b - h_D = h_{f2}$$

$$8 - h_D = \frac{f_2 L_2 Q_2^2}{3d_2^5} = 84280\, Q_2^2$$

and continuity would have been $Q_1 + Q_2 = Q_3$.

If you then attempted to solve this you would soon see that there is no solution.