

An alternative method to solve the above problem is shown below. It does not solve the head at the junction, instead directly solves for a velocity (it may be easily amended to solve for discharge Q)

[For this particular question the method shown above is easier to apply – but the method shown below could be seen as more general as it produces a function that could be solved by a numerical method and so may prove more convenient for other similar situations.]

Again for this we will assume the flow will be from reservoir A to junction D then from D to reservoirs B and C. There are three unknowns u_1 , u_2 and u_3 the three equations we need to solve are obtained from A to B then A to C and from continuity at the junction D.

Flow from A to B

$$\frac{p_A}{\rho g} + \frac{u_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{u_B^2}{2g} + z_B + \frac{4fL_2u_1^2}{2gd_1} + \frac{4fL_2u_2^2}{2gd_2}$$

Putting $p_A = p_B$ and taking u_A and u_B as negligible, gives

$$z_A - z_B = \frac{4fL_2u_1^2}{2gd_1} + \frac{4fL_2u_2^2}{2gd_2}$$

Put in the numbers from the question

$$16 = \frac{4 \times 0.01 \times 120u_1^2}{2g0.12} + \frac{4 \times 0.01 \times 60u_2^2}{2g0.075}$$

$$16 = 2.0387u_1^2 + 1.6310u_2^2$$

(equation i)

Flow from A to C

$$\frac{p_A}{\rho g} + \frac{u_A^2}{2g} + z_A = \frac{p_C}{\rho g} + \frac{u_C^2}{2g} + z_C + \frac{4fL_2u_1^2}{2gd_1} + \frac{4fL_3u_3^2}{2gd_3}$$

Putting $p_A = p_C$ and taking u_A and u_C as negligible, gives

$$z_A - z_C = \frac{4fL_2u_1^2}{2gd_1} + \frac{4fL_3u_3^2}{2gd_3}$$

Put in the numbers from the question

$$24 = \frac{4 \times 0.01 \times 120u_1^2}{2g0.12} + \frac{4 \times 0.01 \times 40u_3^2}{2g0.060}$$

$$24 = 2.0387u_1^2 + 1.3592u_3^2$$

(equation ii)

From continuity at the junction

$$\text{Flow A to D} = \text{Flow D to B} + \text{Flow D to C}$$

$$Q_1 = Q_2 + Q_3$$

$$\frac{\pi d_1^2}{4} u_1 = \frac{\pi d_2^2}{4} u_2 + \frac{\pi d_3^2}{4} u_3$$

$$u_1 = \left(\frac{d_2}{d_1} \right)^2 u_2 + \left(\frac{d_3}{d_1} \right)^2 u_3$$

with numbers from the question

$$u_1 - 0.3906u_2 - 0.25u_3 = 0$$

(equation iii)

the values of u_1 , u_2 and u_3 must be found by solving the simultaneous equation i, ii and iii. The technique to do this is to substitute for equations i, and ii in to equation iii, then solve this expression. It is usually done by a trial and error approach.

i.e. from i,

$$u_2 = \sqrt{9.81 - 1.25u_1^2}$$

from ii,

$$u_3 = \sqrt{17.657 - 1.5u_1^2}$$

substituted in iii gives

$$u_1 - 0.3906\sqrt{9.81 - 1.25u_1^2} - 0.25\sqrt{17.657 - 1.5u_1^2} = 0 = f(u_1)$$

This table shows some trial and error solutions

u	f(u)
1	-1.14769
2	0.289789
1.8	-0.03176
1.85	0.046606
1.83	0.015107
1.82	-0.00057

Giving $u_1 = 1.82$ m/s, so $u_2 = 2.38$ m/s, $u_3 = 12.69$ m/s

Flow rates are

$$Q_1 = \frac{\pi d_1^2}{4} u_1 = 0.0206 \text{ m}^3 / \text{s}$$

$$Q_2 = \frac{\pi d_2^2}{4} u_2 = 0.0105 \text{ m}^3 / \text{s}$$

$$Q_3 = \frac{\pi d_3^2}{4} u_3 = 0.0101 \text{ m}^3 / \text{s}$$

Check for continuity at the junction

$$Q_1 = Q_2 + Q_3$$

$$0.0206 = 0.0105 + 0.0101$$