Example

A petroleum fraction is pumped 2 km from a distillation plant to storage take through a mild steel pipeline, 150 mm I.D. at 0.04 m^3/s rate. What is the pressure drop along the pipe and the power supplied to the pumping unit if it has an efficiency of 50%. The pump impeller is eroded and the pressure at its delivery falls to one half. By how much is the flow rate reduced? Take that: sp.gr. = 0.705, μ= 0.5 m Pa.s e = 0.004 mm.

Solution:

$u = Q/A = (0.04\ m^3/s)/(\pi/4 \times 0.15^2) \Rightarrow u = 2.26\ m/s$

$Re = (705 \times 2.26 \times 0.15)/\ 0.5\times10^{-3} = 4.78\times10^5$

$e/d = 0.000027 \Rightarrow$ Figure (3.7) $f = 2\ \Phi \Rightarrow f = 0.0033$

$\boxed{-\Delta P_{Fs} = \left[4 f\left(\frac{L}{d}\right)\right]\frac{\rho u^2}{2}}$ $= 4\ (0.0033)\ (2000/0.15)\ (705 \times 2.26^2/2) = 316876$ Pa.

Power $= \frac{Q(-\Delta P)}{\eta} = (0.04\ m^3/s)(316876\ Pa)/0.5 = 25.35$ kW

Due to impeller erosion $(-\Delta P)_{new} = (-\Delta P)_{old}/2 = 316876\ Pa/2 = 158438$ Pa

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2) = [(158438)/(2000)][(1000)(0.15)^3/(4)(0.5\times10^{-3})^2] = 1.885\times10^8$

$e/d = 0.000027 \Rightarrow$ From Figure (3.8) $Re = 3 \times10^5 \Rightarrow u = 1.42 m/s$

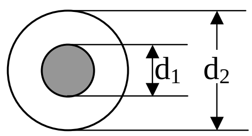
The new volumetric flow rate is now $Q = 1.42\ (\pi/4 \times 0.15^2) = 0.025\ m^3/s$.

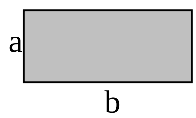
4.9 Friction Losses in Noncircular Conduits

The friction loss in long straight channels or conduits of noncircular cross-section can be estimated by using the same equations employed for circular pipes if the diameter in the Reynolds number and in the friction factor equation is taken as equivalent diameter. The equivalent diameter De or hydraulic diameter defined as four times the cross-sectional area divided by the wetted perimeter of the conduit.

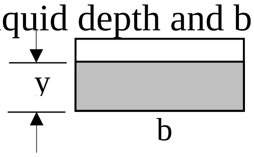
$$De = 4\frac{\text{Cross - sectional area of channel}}{\text{Wetted perimeter of channel}}$$

- For circular cross section.
  $De = 4\ (\pi/4 \times d^2)/\ \pi\ d = d$
- For an annular space with outside diameter d_1 and inside d_2.
  $De = 4\ [\pi/4 \times (d_1^2 - d_2^2)]/\ \pi\ (d_1 + d_2) = d_1+d_2$





- For a rectangular duct of sides a and b.
  $De = 4\ (a.b)\ /\ 2(a + b) = 2ab\ /\ (a + b)$
- For open channels and pattly filled ducts of y-liquid depth and b width
  $De = 4\ (b.y)\ /\ (b + 2y)$



4.10 Selection of Pipe Sizes

In large or complex piping systems, the optimum size of pipe to use for a specific situation depends upon *the relative costs of capital investment*, *power*, *maintenance*, and so on. Charts are available for determining these optimum sizes. However, for small

installations approximations are usually sufficient accurate. A table of representative values of ranges of velocity in pipes is shown in the following table: -

<table>
<tr><th rowspan="2">Type of fluid</th><th rowspan="2">Type of flow</th><th colspan="2">Velocity</th></tr>
<tr><th>ft/s</th><th>m/s</th></tr>
<tr><td rowspan="2">Nonviscous liquid</td><td>Inlet to pump</td><td>2 - 3</td><td>0.6 – 0.9</td></tr>
<tr><td>Process line or Pump discharge</td><td>5 - 8</td><td>1.5 – 2.5</td></tr>
<tr><td rowspan="2">Viscous liquid</td><td>Inlet to pump</td><td>0.2 – 0.8</td><td>0.06 – 0.25</td></tr>
<tr><td>Process line or Pump discharge</td><td>0.5 - 2</td><td>0.15 – 0.6</td></tr>
<tr><td colspan="2">Gas</td><td>30 - 120</td><td>9 – 36</td></tr>
<tr><td colspan="2">Steam</td><td>30 - 75</td><td>9 – 23</td></tr>
</table>

4.11 The Boundary Layer

When a fluid flow over a surface, that part of the stream, which is close to the surface, suffers a significant retardation, and a velocity profile develops in the fluid. In the bulk of the fluid away from the boundary layer the flow can be adequately described by the theory of ideal fluids with zero viscosity (μ =0). However in the thin boundary layer, viscosity is important.

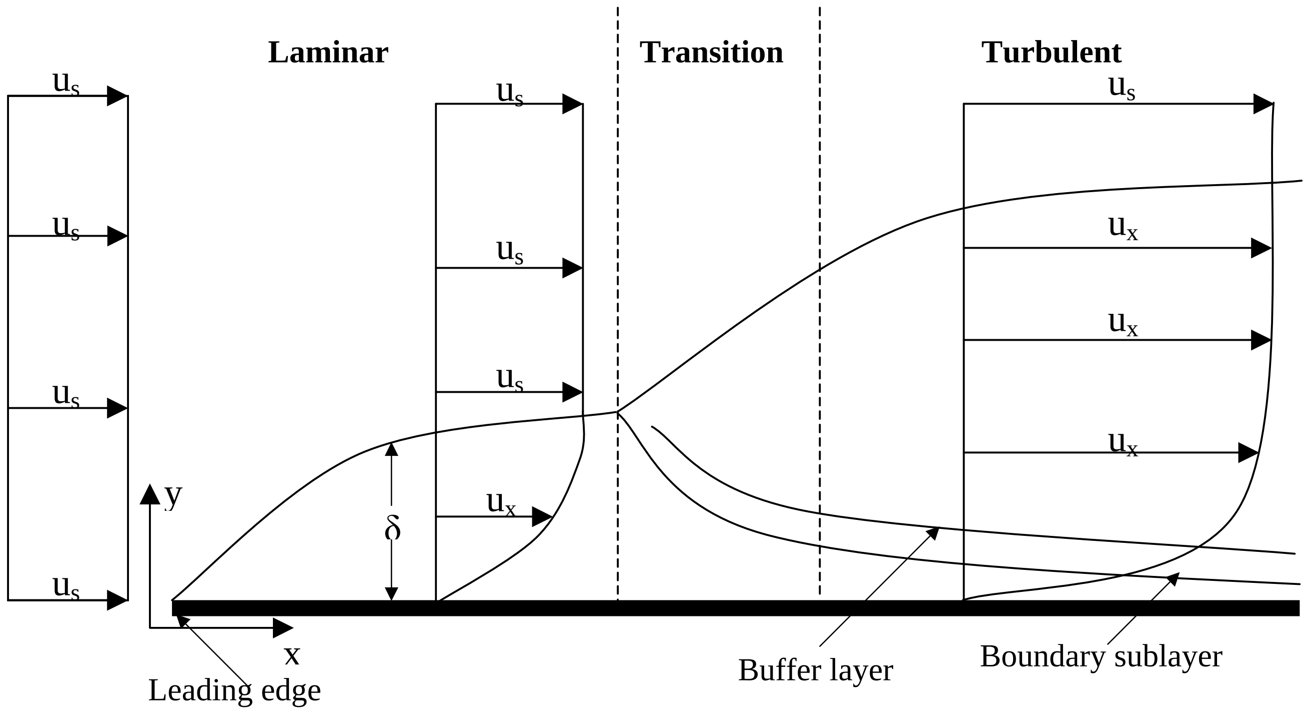


Figure of boundary layer for flow past a flat plate

If the velocity profile of the entrance region of a tube is flat, a certain length of the tube is necessary for the velocity profile to be fully established (developed). This length for the establishment of fully developed flow is called “entrance length”.

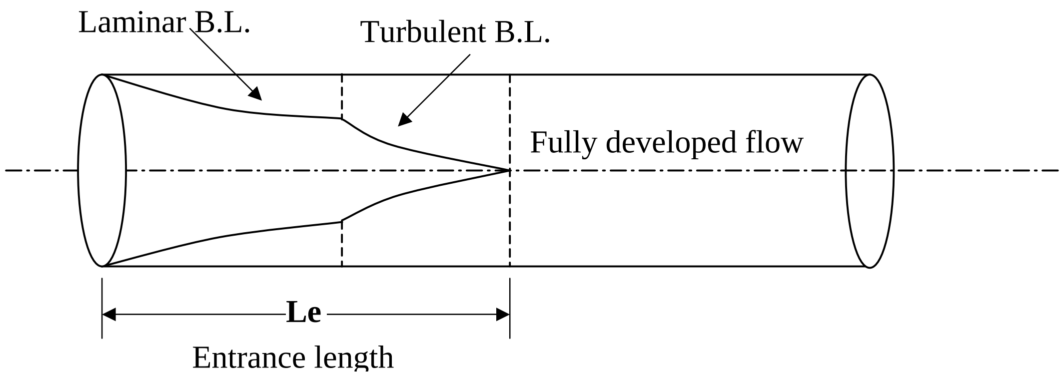


Figure of conditions at entry to pipe.

At the entrance the velocity profile is flat; i.e. the velocity is the same at all positions. As the fluid progresses down the tube, the boundary layer thickness increases until finally they meet at the centerline of the pipe.

For fully developed velocity profile to be formed in laminar flow, the approximate entry length (Le) of pipe having diameter d, is: -

Le/d = 0.0575 Re --------------------laminar

For fully developed velocity profile to be formed in turbulent flow, no relation is available to predict the entry length. As an approximation the entry length (Le) is after 50 diameters downstream of pipe. Thus;

Le/d = 50 ------------------turbulent

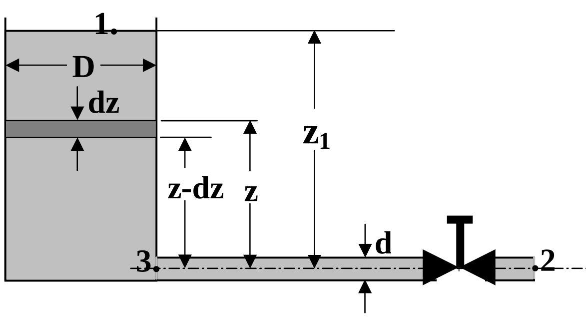
4.12 Unsteady State Problems

Example

A cylindrical tank, 5 m in diameter, discharges through a horizontal mild steel pipe 100 m long and 225 mm diameter connected to the base of the tank. Find the time taken for the water level in the tank to drop from 3 m to 0.3 m above the bottom. The viscosity of water is 1.0 mNs/m^2, e = 0.05 mm.

Solution:

$$\cancel{\frac{\Delta P}{\rho\, g}} + \Delta z + \frac{\Delta u^2}{2\alpha\, g} - \cancel{\frac{\eta\, Ws}{g}} + h_F = 0$$



$u_1 \approx 0$, $z_2 = 0$ (datum line)

at time = 0 $\Rightarrow z = z_1$

at time = t $\Rightarrow z = z$

$$\Rightarrow z_1 = \frac{u_2^2}{2\alpha_2} + h_F \quad \text{at t = 0}$$

$$\boxed{h_{Fs} = \left[4 f\left(\frac{L}{d}\right)\right]\frac{u^2}{2g}} = 90.61\, f\, u_2^2$$

at time = 0 $\Rightarrow z_1 = (0.051 + 90.61\,f)\,u_2^2 \;\Rightarrow u_2 = \sqrt{\dfrac{z_1}{0.051+90.61f}}$

at time = t, $\Rightarrow z = (0.051 + 90.61\,f)\,u_2^2 \;\Rightarrow u_2 = \sqrt{\dfrac{z}{0.051+90.61f}}$ --------------------- (1)

Let the level of liquid in the tank at time (t) is (z)
and the level of liquid in the tank at time (t+dt) is (z–dz)
The volume of liquid discharge during (time =t) to (time = t+dt) is (– dV)

$= (\pi/4\ D^2)\,[z-(z-dz)] = (19.63\ dz)\ m^3$

$Q = dV/dt = -19.63\ (dz/dt)\ m^3/s$ ---(2)

But $Q = A\,u_2 = (\pi/4\ d^2)\,u_2 = (0.04\ m^2)\,u_2$ ------------------------------------(3)

Substitute eq.(1) into eq.(3) to give;

$$Q = 0.04\sqrt{\frac{z}{0.051+90.61f}} \qquad \text{----------------------------------(4)}$$

The equalization between eq.(2) and eq.(4) gives;

$$Q = -19.63\frac{dz}{dt} = 0.04\sqrt{\frac{z}{0.051+90.61f}} \;\Rightarrow \int_0^T dt = \int_3^{0.3} -490.75\sqrt{0.051+90.61f}\,z^{-\frac{1}{2}}dz$$

$$\Rightarrow T = 490.75\sqrt{0.051+90.61f}\int_{0.3}^{3} z^{-\frac{1}{2}}dz \;=490.75\sqrt{0.051+90.61f}\,\left.\frac{z^{\frac{1}{2}}}{1/2}\right|_{0.3}^{3}$$

$$\Rightarrow T = 1169.4\sqrt{0.051+90.61f}$$

$P_3 = P_o + z\rho g$, and $P_2 = P_o$

$\Rightarrow (P_3 - P_2) = (-\Delta P_{Fs})$ the pressure drop along the pipe due to friction

From applied the modified Bernoulli's equation between 3 and 2 $\Rightarrow (-\Delta P_{Fs}) = z\rho g$

But $\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2) = [(z\rho g)/(L)][(\rho d^3/4\mu^2)] = 2.79\times10^8 z$

at z = 3.0 m $\Rightarrow \Phi Re^2 = 8.79\times10^8$

at z = 0.3 m $\Rightarrow \Phi Re^2 = 8.38\times10^7$

e/d = 0.0002 $\Rightarrow$ From Figure (3.8)

$\Rightarrow$at z = 3.0 m Re = 7.0×10^5

$\Rightarrow$at z = 0.3 m Re = 2.2×10^5 —— Turbulent

e/d = 0.0002 $\Rightarrow$ From Figure (3.7)

$\Rightarrow$at z = 3.0 m Re = 7.0×10^5 $\Rightarrow f = 0.0038$

$\Rightarrow$at z = 0.3 m Re = 2.2×10^5 $\Rightarrow f = 0.004$

taking a value of $f = 0.004$, and assume it constant

$\therefore$ T = 752 s