Example -

Two tanks, the bottoms of which are at the same level, are connected with one another by a horizontal pipe 75 mm diameter and 300 m long. The pipe is bell-mouthed at each end so that losses on entry and exit are negligible. One tank is 7 m diameter and contains water to a depth of 7 m. The other tank is 5 m diameter and contains water to a depth of 3 m. If the tanks are connected to each other by means of the pipe, how long will it take before the water level in the larger tank has fallen to 6 m? Take e =0.05 mm.

Solution:

At any time (t) the depth in the larger tank is (h) and the depth in the smaller tank is (H)

$$\cancel{\frac{\Delta P}{\rho g}} + \Delta z + \cancel{\frac{\Delta u^2}{2\alpha g}} - \cancel{\frac{\eta Ws}{g}} + h_F = 0$$

$\Rightarrow$ $\Delta z = h - H = h_{Fs}$ -----------------------------------(1)

When the level in the large tank fall to (h), the level in the small tank will rise by a height (x) by increasing to reach a height (H).

The volume of the liquid in large tank that discharged to small tank is;

$= \pi/4 \times 7^2 (7–h) = 38.48 (7–h)\ m^3$

and is equal to $= \pi/4 \times 5^2 (x) = 38.48 (7–h)$

$\Rightarrow$ $x = 13.72 – 1.96 h$ ---------------------------(2)

$H = 3 + x = 3 + 13.72 – 1.96 h$

$\Rightarrow$ $H = 16.72 – 1.96 h$ ---------------------------(3)

Substitute eq.(3) into eq.(1), to give,

$h – (16.72 – 1.96 h) = h_{Fs} = \left[4 f\left(\frac{L}{d}\right)\right]\frac{u^2}{2g}$

$2.96 h – 16.72 = 815.5 f u^2 \quad \Rightarrow u = \sqrt{\frac{0.00363\, h - 0.02}{f}}$

The level of water in the large at (t = 0) = 7 m

The level of water in the large at (t = t) = h m

The level of water in the large at (t = t+dt) = (h–dh) m

The discharge of liquid during the timed (dt) is,

$Q = dV / dt = \pi/4\ 7^2 [h–(h–dh)] / dt = \pi/4\ 7^2 (dh / dt)$ --------------------(4)

But $Q = A u = \pi/4\ d^2 \Rightarrow Q = \frac{\pi}{4}(0.075)^2\sqrt{\frac{0.00363\, h - 0.02}{f}}$ ------------------------(5)

By equalization between eq.(4) and eq.(5) $\Rightarrow dt = -8711.11\frac{dh}{\sqrt{\frac{0.00363 - 0.02}{f}}}$

e/d = 0.05/75 = 0.00067 assume f = 0.004

$$\int_0^T dt = -8711.11\int_7^6 (0.9h-5)^{-0.5}\,dh = 8711.11\left(\frac{1}{0.9}\right)\frac{(0.9h-5)^{0.5}}{0.5}\Bigg|_6^7 = 19358(1.3^{0.5}-0.4^{0.5})$$

$\Rightarrow$ T = 9777.67 s

Q = [π/4 7^2 (7 – 6)] / 9777.67 = 0.00393 m^3/s average volumetric flow rate

u = Q / A = (0.00393 m^3/s) / (π/4 x 0.075^2) = 0.89 m/s ⇒ Re = 6.6552 x10^4

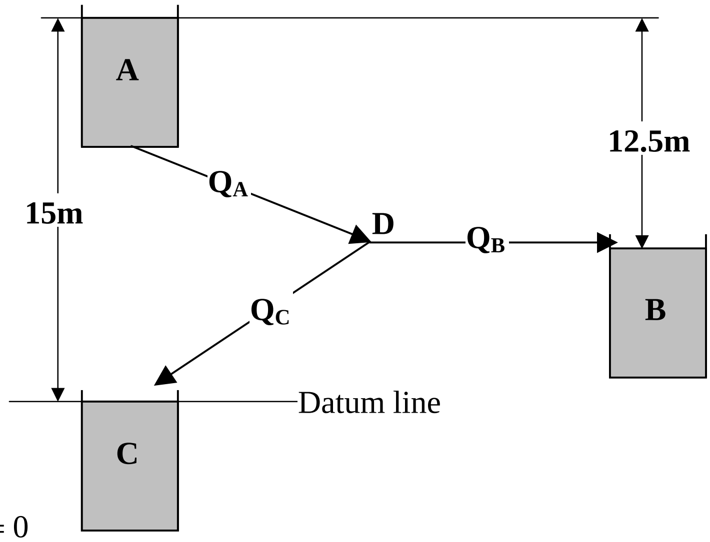
e/d = 0.00067 ⇒ From Figure (3.7) ⇒ f =0.006

Repeat the integration based on the new value of (f = 0.006) ⇒ T = 9777.67 s

Example -

Water is being discharged, from a reservoir, through a pipe 4 km long and 50 cm I.D. to another reservoir having water level 12.5 m below the first reservoir. It is required to feed a third reservoir, whose level is 15 m below the first reservoir, through a pipe line 1.5 km long to be connected to the pipe at distance of 1.0 km from its entrance. Find the diameter of this new pipe, so that the flow into both the reservoirs may be the same.

Solution:



AD + DB = 4,000 m, its i.d = 50 cm

AD = 1,000 m ⇒ DB = 3,000 m

DC = 1,500 m

$Q_A = Q_B + Q_C$

$Q_B = Q_C = Q_A/2$-----------------(1)

A-D

$$-\frac{P_D}{\rho} + (z_D - 15)g + 4f\left(\frac{L}{d}\right)_A \frac{u_A^2}{2} = 0$$

$$u = \frac{4Q}{\pi\, d^2}$$

$$-\frac{P_D}{\rho} + z_D g - 15g + 4f\left(\frac{1{,}000}{0.5}\right)\left(\frac{16Q_A^2}{2\pi^2 d_A^4}\right) = 0$$

$$-\frac{P_D}{\rho} + z_D g - 147.1 + 830\, Q_A^2 = 0 \text{ ------------------------(2)}$$

A-B

$$-\frac{P_D}{\rho} - (z_D - 15)g + 4f\left(\frac{3{,}000}{0.5}\right)\left(\frac{16Q_B^2}{2\pi^2 d_B^4}\right) = 0$$

$$-\frac{P_D}{\rho} - z_D g + 24.5 + 2{,}490\, Q_B^2 = 0 \text{ ------------------------(3)}$$

A-B

$$-\frac{P_D}{\rho} - z_D g + 4f\left(\frac{1{,}500}{d_C}\right)\left(\frac{16Q_C^2}{2\pi^2 d_C^4}\right) = 0$$

$$-\frac{P_D}{\rho} - z_D g + 38.9\,\frac{Q_C^2}{d_C^5} = 0 \text{ --------------------------------(4)}$$

Substitute eq.(1) into eqs.(3) and (4)

Equation (2) $-\frac{P_D}{\rho} + z_D g - 147.1 + 830\, Q_A^2 = 0$ -------------------------(2)

Equation (3) $-\frac{P_D}{\rho} - z_D g + 24.5 + 622.5\, Q_A^2 = 0$ -------------------------(5)

Equation (4) $-\frac{P_D}{\rho} - z_D g + 9.72\, \frac{Q_A^2}{d_C^5} = 0$ --------------------------------(6)

eq.(2) + eq.(5) ⇒ – 122.6 + 1452.5 $Q_A^2 = 0$ ⇒ $Q_A = 0.29\ m^3/s$

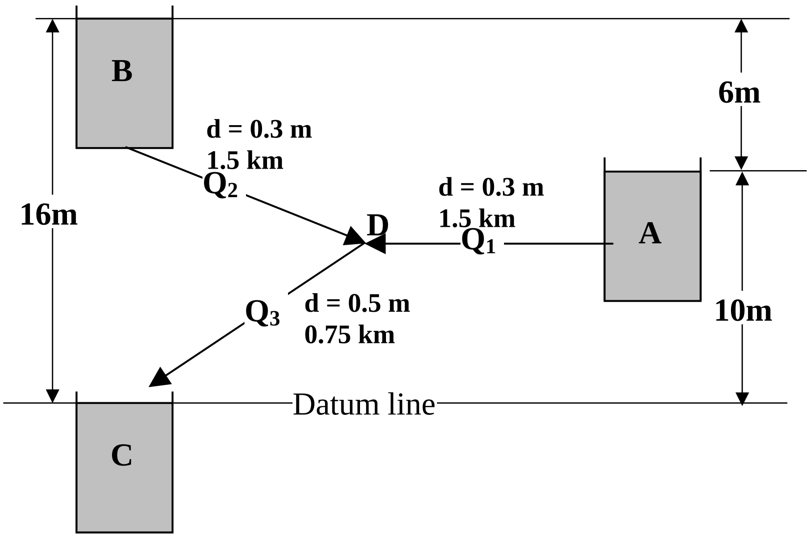
⇒ $Q_B = Q_C = (0.29\ m^3/s) / 2 = 0.145\ m^3/s$

eq.(5) – eq.(6) ⇒ $24.5 + 622.5(0.29)^2 - 9.72(0.29)^2 / d_C^5 = 0$ ⇒ $d_C^5 = 0.0106\ m^5$

⇒ $d_C = 0.4\ m = 40\ cm$.

Example

Two storage tanks, A and B, containing a petroleum product, discharge through pipes each 0.3 m in diameter and 1.5 km long to a junction at D, as shown in Figure. From D the liquid is passed through a 0.5 m diameter pipe to a third storage tank C, 0.75 km away. The surface of the liquid in A is initially 10 m above that in C and the liquid level in B is 6 m higher than that in A. Calculate the initial rate of discharge of liquid into tank C assuming the pipes are of mild steel. The density and viscosity of the liquid are 870 kg/m^3 and 0.7 m Pa.s respectively.



Solution:

Because the pipes are long, the kinetic energy of the fluid and minor losses at the entry to the pipes may be neglected. It may be assumed, as a first approximation, that *f* is the same in each pipe and that the velocities in pipes AD, BD, and DC are u_1, u_2, and u_3 respectively, if the pressure at D is taken as P_D and point D is z_D m above the datum for the calculation of potential energy, the liquid level in C.

Then applying the energy balance equation between D and the liquid level in each of the tanks gives:

A-D $$-\frac{P_D}{\rho} + (z_D - 10)g + 4f\left(\frac{1{,}500}{0.3}\right)\frac{u_1^2}{2\alpha_1} = 0$$

B-D $$-\frac{P_D}{\rho} + (z_D - 16)g + 4f\left(\frac{1{,}500}{0.3}\right)\frac{u_2^2}{2\alpha_2} = 0$$

D-C $$-\frac{P_D}{\rho} - (z_D)g + 4f\left(\frac{750}{0.5}\right)\frac{u_3^2}{2\alpha_3} = 0$$

Assume turbulent flow in all pipes

$\Rightarrow$ **A-D** $\quad -\dfrac{P_D}{\rho}+z_D g-98.1+10{,}000 f\ u_1^2=0$ ------------------------------(1)

B-D $\quad -\dfrac{P_D}{\rho}+z_D g-156.96+10{,}000 f\ u_2^2=0$ ----------------------------(2)

D-C $\quad -\dfrac{P_D}{\rho}-z_D g+3{,}000 f\ u_3^2=0$ ------------------------------(3)

eq.(1) – eq.(2) $\Rightarrow$ $58.86 + 10{,}000\, f\,(u_1^2 - u_2^2) = 0$ ----------------------(4)

eq.(2) – eq.(3) $\Rightarrow$ $-156.96 + 10{,}000\, f\,(u_2^2 + 0.3u_3^2) = 0$ ---------------(5)

$Q_1 + Q_2 = Q_3 \Rightarrow [(\pi/4\ 0.3^2)\ u_1] + [(\pi/4\ 0.3^2)\ u_2] = [(\pi/4\ 0.5^2)\ u_3]$

$\Rightarrow u_1 + u_2 = 2.78\ u_3$ ------------------------------------(6)

equations (4), (5), and (6) are three equations with 4 unkowns. As first approximation for e/d = 0.0001 to 0.00017 $\Rightarrow$ $f = 0.004$

$\Rightarrow$ eq.(4) become $58.86 + 40\ (u_1^2 - u_2^2) = 0$ -----------------------------(7)

$\Rightarrow$ eq.(5) become $-156.96 + 40\ (u_2^2 - 0.3u_3^2) = 0$ ----------------------(8)

From eq.(7) $u_1^2 = u_2^2 - 1.47$ --------------------------------(9)

$u_3 = (u_1 + u_2) \Rightarrow u_3^2 = (1/2.78)^2\ (u_1^2 + 2u_1\ u_2 + u_2^2)$

$\Rightarrow u_3^2 = (1/2.78)^2\ [u_2^2 + (u_2^2 - 1.47) + 2u_2(u_2^2 - 1.47)^{0.5}]$

$\Rightarrow u_3^2 = (1/2.78)^2\ [2u_2^2 - 1.47 + 2u_2(u_2^2 - 1.47)^{0.5}]$ -----------------(10)

Substitute eq.(10) into (8)

$\Rightarrow -156.96 + 40\ \{u_2^2 + 0.3(1/2.78)^2\ [2u_2^2 - 1.47 + 2u_2(u_2^2 - 1.47)^{0.5}]\} = 0$

$\Rightarrow u_2(u_2^2 - 1.47)^{0.5} = (159.24 - 43.2\ u_2^2)/\ 3.2$ squaring the two limits

$\Rightarrow u_2^2(u_2^2 - 1.47)^{0.5} = (49.8 - 13.5\ u_2^2)^2$

$\Rightarrow u_2^4 - 7.41u_2^2 + 13.68 = 0$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

$\Rightarrow$ either $u_2^2 = 3.922$ or $u_2^2 = 3.488$

$\Rightarrow u_2 = 1.98$ m/s or $u_2 = 1.87$ m/s

Substituting u_2 into eq.(9)

$\Rightarrow u_1 = 1.56$ m/s or $u_1 = 1.42$ m/s

Substituting u_2, and u_2 into eq.(6)

$\Rightarrow u_3 = 1.3$ m/s or $u_3 = 1.18$ m/s

The lower set values satisfies equation (8)

$\Rightarrow u_1 = 1.42$ m/s, $u_2 = 1.87$ m/s, and $u_3 = 1.18$ m/s

$\Rightarrow Re_1 = 5.3 \times 10^5$, $Re_2 = 6.9 \times 10^5$, and $Re_3 = 7.3 \times 10^5$

From Figure (3.7) $\Rightarrow$ f_1 =0.0043, f_2 =0.0036, and f_3 =0.0038

$\Rightarrow$ the assumption $\underline{f = 0.004 \text{ is ok.}}$

$Q_3 = (\pi/4\ 0.5^2)\ u_3 = (\pi/4\ 0.5^2)\ 1.18 = 0.23\ m^3/s$