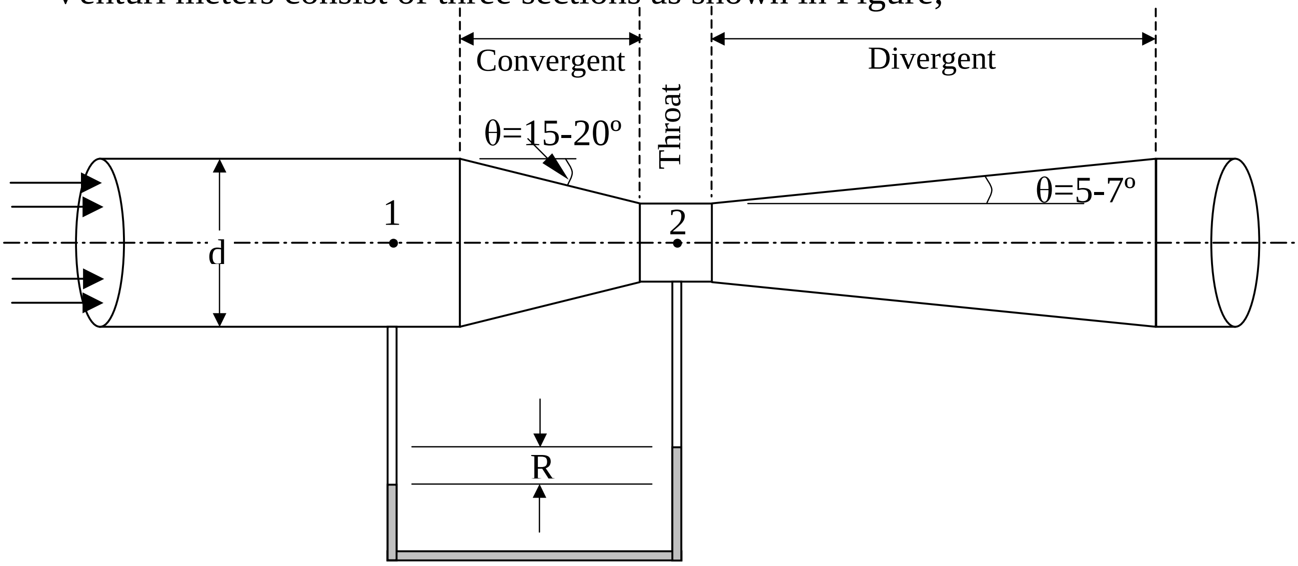
7.2.2 <u>Measurement by Flow Through a Constriction</u>

In measuring devices where the fluid is accelerated by causing it to flow through a constriction, the kinetic energy is thereby increased and the pressure energy therefore decreases. The flow rate is obtained by measuring the pressure difference between the inlet of the meter and a point of reduced pressure.

Venturi meters, ***orifice meters***, and ***flow nozzles*** measure the volumetric flow rate Q or average (mean linear) velocity u. In contrast the Pitot tube measures a point (local) velocity u_x.

7.2.2.1 <u>Venturi Meter</u>

Venturi meters consist of three sections as shown in Figure;



- From continuity equation $A_1u_1 = A_2u_2 \Rightarrow u_1 = (A_2/A_1)\, u_2$
- From Bernoulli's equation between points 1 and 2

$$\frac{P_1}{\rho\, g} + \frac{u_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho\, g} + \frac{u_2^2}{2g} + \cancel{z_2}$$

$$\Rightarrow \frac{P_1 - P_2}{\rho\, g} = \frac{u_2^2 - u_1^2}{2g} = \frac{u_2^2}{2g}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = \frac{u_2^2}{2g}\left[\frac{A_1^2 A_2^2}{A_1^2}\right]$$

$$\Rightarrow \qquad u_2 = \sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)\left[\frac{1}{1-(A_2/A_1)^2}\right]} = \sqrt{\frac{2(-\Delta P)}{\rho}}\,\frac{A_1}{\sqrt{A_1^2 - A_2^2}}$$

or $$u_2 = \sqrt{(2g\Delta h)\left[\frac{1}{1-(A_2/A_1)^2}\right]} = \sqrt{2g\Delta h}\,\frac{A_1}{\sqrt{A_1^2 - A_2^2}}$$

or $$u_2 = \sqrt{\left(\frac{2R(\rho_m - \rho)g}{\rho}\right)\left[\frac{1}{1-(A_2/A_1)^2}\right]} = \sqrt{\frac{2R(\rho_m - \rho)g}{\rho}}\,\frac{A_1}{\sqrt{A_1^2 - A_2^2}}$$

All these equation of velocity ***at throat*** u_2, which derived from Bernoulli's equation are for ideal fluids. Using a coefficient of discharge C_d to take account of the frictional losses in the meter and of the parameters of kinetic energy correction α_1 and α_2. Thus the volumetric flow rate will be obtained by: -

$$Q = u_2 A_2 = C_d \sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)\left[\frac{A_2^2}{1-(A_2/A_1)^2}\right]} = C_d \sqrt{\frac{2(-\Delta P)}{\rho}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

or $$Q = C_d \sqrt{(2g\Delta h)\left[\frac{A_2^2}{1-(A_2/A_1)^2}\right]} = C_d \sqrt{2g\Delta h} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

or $$Q = C_d \sqrt{\left(\frac{2R(\rho_m - \rho)g}{\rho}\right)\left[\frac{A_2^2}{1-(A_2/A_1)^2}\right]} = C_d \sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$\boxed{\dot{m} = Q\,\rho, \qquad G = \rho\, u = \frac{\dot{m}}{A}}$$

For many meters and for Re > 10^4 at point 1

$C_d = 0.98$ for $d_1 < 20$ cm

$C_d = 0.99$ for $d_1 > 20$ cm

Example -7.5-

A horizontal Venturi meter with d_1 = 20 cm, and d_2 = 10 cm, is used to measure the flow rate of oil of sp.gr. = 0.8, the discharge through venture meter is 60 lit/s. find the reading of (oil-Hg) differential Take C_d = 0.98.

Solution:

$Q = u_2 A_2$ = 60 lit/s (m^3/1000lit) = 0.06 m^3/s

$$0.06 = C_d \sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} = 0.98\sqrt{\frac{2R(13600-800)9.81}{800}} \frac{(\pi/4)^2(0.1)^2(0.2)^2}{\sqrt{(\pi/4)^2[(0.2)^4 - (0.1)^4]}}$$

$\Rightarrow$ R = 0.1815 m Hg = 18.15 cm Hg

Example -7.6-

A horizontal Venturi meter is used to measure the flow rate of water through the piping system of 20 cm I.D, where the diameter of throat in the meter is d_2 = 10 cm. The pressure at inlet is 17.658 N/cm^2 gauge and the vacuum pressure of 35 cm Hg at throat. Find the discharge of water. Take C_d = 0.98.

Solution:

P_1 = 17.658 N/cm^2 (100 cm / m)^2 = 176580 Pa

P_2 = –35 mm Hg (m / 100 cm) 9.81 (13600) = –46695.6 Pa

$P_1 - P_2$ = 176580–(–46695.6) = 223275.6 Pa

$$Q = u_2 A_2 = C_d \sqrt{\frac{2\Delta P}{\rho}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} = 0.98\sqrt{\frac{2(223275.6)}{1000}} \frac{(0.2)^2[(\pi/4)(0.1)^2]}{\sqrt{[(0.2)^4 - (0.1)^4]}}$$

$\Rightarrow$ Q = 0.168 m^3/s