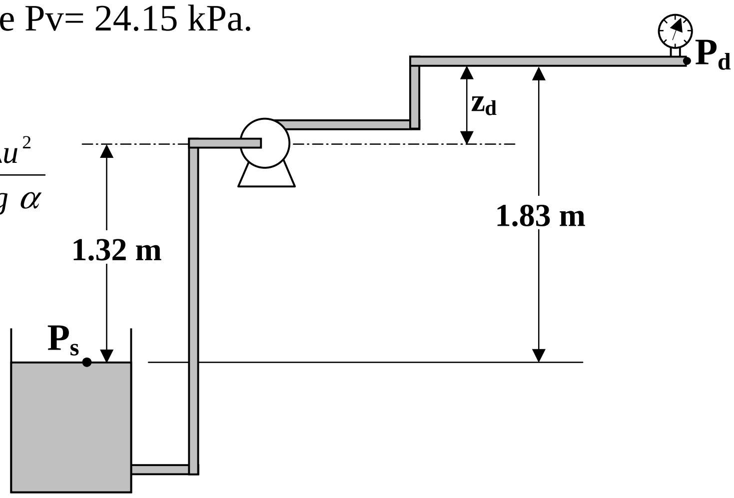
**Example -5.1-**

A petroleum product is pumped at a rate of 2.525 x 10-3 $m^3/s$ from a reservoir under atmospheric pressure to 1.83 m height. If the pump 1.32 m height from the reservoir, the discharge line diameter is 4 cm and the pressure drop along its length 3.45 kPa. The gauge pressure reading at the end of the discharge line 345 kPa. The pressure drop along suction line is 3.45 kPa and pump efficiency η=0.6 calculate:-

**(i)** The total head of the system Δh. **(ii)** The power required for pump. **(iii)** The NPSH

Take that: the density of this petroleum product ρ=879 $kg/m^3$, the dynamic viscosity μ=6.47 x 10-4 Pa.s, and the vapor pressure Pv= 24.15 kPa.




**Solution:**

**(i)**

$$\Delta h = (z_d - z_s) + \left(\frac{P_d - P_s}{\rho\, g}\right) + [(h_F)_d + (h_F)_s] + \frac{\Delta u^2}{2g\,\alpha}$$

$u_s = 0$

$u_d = (2.525 \times 10^{-3}\ m^3/s)/(\pi/4\ 0.04^2)$

$= 2$ m/s

$Re_d = (879 \times 2 \times 0.04)/\ 6.47 \times 10^{-4}$

$= 1.087 \times 10^5$

The pressure drop in suction line 3.45kPa

$\Rightarrow (h_F)_s = 3.45 \times 10^3/(879 \times 9.81)$

$= 0.4$ m

And in discharge line is also 3.45 kPa $\Rightarrow (h_F)_d = 0.4$ m

The kinetic energy term = $2^2/(2 \times 9.81)$ =0.2 m

The pressure at discharge point = gauge + atmospheric pressure = 345 + 101.325

= 446.325 kPa

The difference in pressure head between discharge and suction points is

$(446.325 - 101.325) \times 10^3\ /(879 \times 9.81) = 40$ m

Δz = 1.83 m

$\Rightarrow \Delta h = 40\text{ m} + 1.83\text{ m} + 0.2\text{ m} + 0.4\text{ m} + 0.4\text{ m} = 42.83$ m

**(ii)**

$$P = \frac{Q\Delta P}{\eta} = \frac{Q\Delta h\ \rho\ g}{\eta} = [(2.525 \times 10^{-3}\ m^3/s)(42.83\ m)(879\ kg/m^3)(9.81\ m/s^2)]/0.6$$

$\Rightarrow$ P = 1.555 kW

**(iii)**

$$NPSH = z_s + \left(\frac{P_s - P_v}{\rho\, g}\right) - (h_F)_s$$

$= (-1.32) + (1.01325 \times 10^5 - 24150)/\ (879 \times 9.81) - 0.4$ m

$= 7.23$ m

## 5.9 <u>Centrifugal Pump Relations</u>

The power ($P_E$) required in ***an ideal centrifugal pump*** can be expected to be a function of the liquid density ($\rho$), the impeller diameter (D), and the rotational speed of the impeller (N). If the relationship is assumed to be given by the equation,

$$P_E = c\,\rho^a N^b D^c \qquad \text{----------------------------------}(1)$$

then it can be shown by dimensional analysis that

$$P_E = c_1\,\rho\,N^3 D^5 \qquad \text{----------------------------------}(2)$$

where, $c_1$ is a constant which depends on the geometry of the system.

The power ($P_E$) is also proportional to the product of the volumetric flow rate (Q) and the total head ($\Delta h$) developed by the pump.

$$P_E = c_2\, Q\, \Delta h \qquad \text{---------------------------------(3)}$$

where, $c_2$ is a constant.

The volumetric flow rate (Q) and the total head ($\Delta h$) developed by the pump are: -

$$Q = c_3\, N\, D^3 \qquad \text{---------------------------------(4)}$$

$$\Delta h = c_4\, N^2\, D^2 \qquad \text{---------------------------------(5)}$$

where, $c_3$ and $c_4$ are constants.

Equation (5) could be written in the following form,

$$\Delta h^{3/2} = c_4^{3/2}\, N^3\, D^3 \qquad \text{---------------------------------(6)}$$

Combine equations (4) and (6) [ eq. (4) divided by eq. (6)] to give;

$$\frac{Q}{\Delta h^{\frac{3}{2}}} = \frac{c_3}{c_4^{\frac{3}{2}}}\frac{1}{N} \Rightarrow \frac{QN^2}{\Delta h^{\frac{3}{2}}} = const. \qquad \text{------------------------------(7)}$$

or, $$\frac{N\sqrt{Q}}{\Delta h^{\frac{3}{4}}} = const. = N_s \qquad \text{---------------------------------(8)}$$

When the rotational speed of the impeller N is (rpm), the volumetric flow rate Q in (USgalpm) and the total head $\Delta h$ developed by the pump is in (ft), the constant $N_s$ in equation (8) is known as ***the specific speed of the pump***. The specific speed is used as an index of pump types and always evaluated at the best efficiency point (bep) of the pump. Specific speed vary in the range (400 – 10,000) depends on the impeller type, and has the dimensions of $(L/T^2)^{3/4}$. [ British gal=1.2USgal, $ft^3$=7.48USgal, $m^3$=264USgal]

### 5.9.1 Homologous Centrifugal Pumps

Two different size pumps are said to be geometrically similar when the ratios of corresponding dimensions in one pump are equal to those of the other pump. Geometrically similar pumps are said to be homologous. A sets of equations known as the ***affinity laws*** govern the performance of homologous centrifugal pumps at various impeller speeds.

For the tow homologous pumps, equations (4), and (5) are given

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3 \qquad \text{---------------------------------(9)}$$

$$\frac{\Delta h_1}{\Delta h_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 \qquad \text{---------------------------------(10)}$$

Similarly for the tow homologous pumps equation (2)can be written in the form;

$$\frac{P_{E1}}{P_{E2}} = \left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^5 \qquad \text{---------------------------------(11)}$$

And by analogy with equation (10),

$$\frac{NPSH_1}{NPSH_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 \text{---------------------------------(12)}$$

Equations (9), (10), (11), and (12) are the affinity law for homologous centrifugal pumps.