Dimensional Analysis

Introduction

Any phenomenon is physical sciences and engineering can be described by the *fundamentals dimensions* mass, length, time, and temperature. Till the rapid development of science and technology the engineers and scientists depend upon the experimental data. But the rapid development of science and technology has created new mathematical methods of solving complicated problems, which could not have been solved completely by analytical methods and would have consumed enormous time. This mathematical method of obtaining the equations governing certain natural phenomenon by balancing the fundamental dimensions is called *(Dimensional Analysis)*. Of course, the equation obtained by this method is known as *(Empirical Equation)*.

Fundamentals Dimensions

The various physical quantities used by engineer and scientists can be expressed in terms of fundamentals dimensions are: Mass (M), Length (L), Time (T), and Temperature (θ). All other quantities such as area, volume, acceleration, force, energy, etc., are termed as " derived quantities".

Dimensional Homogeneity

An equation is called *"dimensionally homogeneous"* if the fundamentals dimensions have identical powers of [L T M] (i.e. length, time, and mass) on both sides. Such an equation be independent of the system of measurement (i.e. metric, English, or S.I.). Let consider the common equation of volumetric flow rate,

$$Q = A\ u$$

$$L^3T^{-1} = L^2\,LT^{-1} = L^3T^{-1}.$$

We see, from the above equation that both right and left hand sides of the equation have the same dimensions, and the equation is therefore dimensionally homogeneous.

Methods of Dimensional Analysis

Dimensional analysis, which enables the variables in a problem to be grouped into form of *dimensionless groups*. Thus reducing the effective number of variables. The method of dimensional analysis by providing a smaller number of independent groups is most helpful to experimenter.

Many methods of dimensional analysis are available; two of these methods are given here, which are:

1- **Rayleigh's method (or Power series)**

2- **Buckingham's method (or Π-Theorem)**

(Rayleigh's method (or Power series

In this method, the functional relationship of some variable is expressed in the form of an exponential equation, which must be dimensionally homogeneous. If (y) is some function of independent variables (x_1, x_2, x_3, ……..etc.), then functional relationship may be written as;

$$y = f(x_1, x_2, x_3, \ldots\ldots\text{etc.})$$

The dependent variable (y) is one about which information is required; whereas the independent variables are those, which govern the variation of dependent variables.

The Rayleigh's method is based on the following steps:-

1- First of all, write the functional relationship with the given data.

2- Now write the equation in terms of a constant with exponents i.e. powers a, b, c,...

3- With the help of the principle of dimensional homogeneity, find out the values of a, b, c, … by obtaining simultaneous equation and simplify it.

4- Now substitute the values of these exponents in the main equation, and simplify it.

Example

If the capillary rise (h) depends upon the specific weight (sp.wt) surface tension (σ) of the liquid and tube radius (r) show that:

$$h = r\phi\left(\frac{\sigma}{(sp.wt.)\, r^2}\right), \text{ where } \phi \text{ is any function.}$$

Solution:

Capillary rise (h) m $\equiv [L]$

Specific weight (sp.wt) N/m^3 $(MLT^{-2}\, L^{-3}) \equiv [ML^{-2}T^{-2}]$

Surface tension (σ) N/m $(MLT^{-2}\, L^{-1}) \equiv [MT^{-2}]$

Tube radius (r) m $\equiv [L]$

$$h = f\,(sp.wt., \sigma, r)$$

$$h = k\,(sp.wt.^a, \sigma^b, r^c)$$

$$[L] = [ML^{-2}T^{-2}]^a\, [MT^{-2}]^b [L]^c$$

Now by the principle of dimensional homogeneity, equating the power of M, L, T on both sides of the equation

For M $\quad 0 = a + b \quad \Rightarrow \quad a = -b$

For L $\quad 1 = -2a + c \quad \Rightarrow \quad a = -b$

For T $\quad 0 = -2a - 2b \quad \Rightarrow \quad a = -b$

$$h = k\,(sp.wt.^{-b}, \sigma^b, r^{1-2b})$$

$$h = k\, r\left(\frac{\sigma}{sp.wt.\, r^2}\right)^b \qquad \therefore \quad h = r\phi\left(\frac{\sigma}{(sp.wt.)\, r^2}\right)$$