$$\Rightarrow \quad \boxed{G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{P_2 - P_1}{\upsilon_m} + 4\phi \frac{L}{d} G^2 = 0}$$

If $(P_1 - P_2) / P_1 < 0.2$ the fist term of kinetic energy $[G^2 \ln(P_1/P_2)]$ is negligible.

$$\Rightarrow \quad \boxed{-\Delta P = (P_1 - P_2) = 4\phi \frac{L}{d} G^2 \upsilon_m = 4\phi \frac{L}{d} \rho_m u_m^2 = 4 f \frac{L}{d} \frac{\rho_m u_m^2}{2}}$$

It is used for low-pressure drop.

(i.e. the fluid can be treated as an incompressible fluid at the mean pressure in the pipe.)

8.3.1.1 Maximum Velocity in Isothermal Flow

From equation of isothermal conditions,

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1} + 4\phi \frac{L}{d} G^2 = 0$$

the mass velocity G = 0 when ($P_1 = P_2$)

At some intermediate value of P_2, the flow must therefore be a maximum. To find it, the differentiating the above equation with respect to P_2 for constant P_1 must be obtained.

i.e. ($dG/dP_2 = 0$),

First dividing the above equation by G^2

$$\Rightarrow \frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1} \frac{1}{G^2} + \ln\left(\frac{P_1}{P_2}\right) + 4\phi \frac{L}{d} = 0$$

Then differentiating with respect to P_2

$$\frac{2P_2}{2 P_1 \upsilon_1 G^2} + \frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1} \left(-2G^{-3}\right) \frac{dG}{dP_2} + \frac{1}{P_1 / P_2} \left(\frac{-P_1}{P_2^2}\right) = 0$$

Rearrangement

$$\frac{P_2}{P_1 \upsilon_1 G^2} + \frac{2}{G^3} \left(\frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1}\right) \frac{dG}{dP_2} - \frac{1}{P_2} = 0$$

maximum velocity when ($dG/dP_2 = 0$) where, $P_2 = P_w$, and $G = G_w$

$$\frac{P_w}{P_1 \upsilon_1 G_w^2} = \frac{1}{P_w} \quad \Rightarrow \quad G_w^2 = \frac{P_w^2}{P_1 \upsilon_1}$$

but for isothermal conditions $P_1 \upsilon_1 = P_w \upsilon_w \quad \Rightarrow \quad P_w = P_1 \upsilon_1 / \upsilon_w$

$$\Rightarrow \quad G_w^2 = \frac{P_w}{\upsilon_w} \quad \Rightarrow \quad \left(\frac{u_w}{\upsilon_w}\right)^2 = \frac{P_w}{\upsilon_w}$$

$$\boxed{\therefore \quad u_w = \sqrt{P_w \upsilon_w} = \sqrt{P \upsilon}}$$

i.e. the sonic velocity is the maximum possible velocity.

$$\dot{m}_{\max} = A \rho_w u_w = A \frac{\sqrt{P_w \upsilon_w}}{\upsilon_w} = A \sqrt{\frac{P_w}{\upsilon_w}} \cdots\cdots\cdots \times \sqrt{\frac{P_w}{P_w}}$$

$$\Rightarrow \dot{m}_{\max} = A \sqrt{\frac{P_w^2}{P_w \upsilon_w}} = A P_w \sqrt{\frac{1}{P_w \upsilon_w}} = A P_w \sqrt{\frac{1}{P_1 \upsilon_1}} = A P_w \sqrt{\frac{1}{P_2 \upsilon_2}}$$

To find P_w, the following equation is used,

$$\boxed{\ln\left(\frac{P_1}{P_w}\right)^2 + 1 - \left(\frac{P_1}{P_w}\right)^2 + 8\phi \frac{L}{d} = 0}$$

to get P_w at any given P_1

Example -8.1-

Over a 30 m length of a 150 mm vacuum line carrying air at 295 K, the pressure falls from 0.4 kN/m2 to 0.13 kN/m2. If the relative roughness e/d is 0.003 what is the approximate flow rate? Take that $\mu_{\text{air at 295 K}} = 1.8 \times 10^{-5}$ Pa.s

Solution:

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2\,P_1 \upsilon_1} + 4\phi \frac{L}{d} G^2 = 0$$

It is required the velocity or G for calculating Re that used to estimate Φ from Figure (3.7)-vol.I. i.e. the solution is by **trial and error** technique.

1- Assume Φ = 0.004

$$\upsilon_1 = \frac{1}{\rho_1} = \frac{RT}{P_1 Mwt} = \frac{8.314\,(\text{Pa.m}^3/\text{mol.K})\ 295\text{K}\ [(10^3\ \text{mol/kmol})]}{(0.4\times 10^3\,\text{Pa})\quad 29\ \text{kg/kmol}}$$

$= 211.434$ m^3/kg

$$\Rightarrow G^2 \ln\left(\frac{0.4}{0.13}\right) + \frac{(0.13\times 10^3 - 0.4\times 10^3)}{2(0.4\times 10^3)211.434} + 4(0.004)\frac{30}{0.15} G^2 = 0$$

$\Rightarrow \quad 4.324\, G^2 = 0.846 \Rightarrow \quad G = 0.44$ kg/m^2.s

Re = G d / μ = 3686 ⇒ Φ = 0.005 (Figure 3.7)

2- Assume Φ = 0.005

$\Rightarrow \quad 1.124\, G^2 + 4\, G^2 = 0.846 \Rightarrow \quad G = 0.41$ kg/m^2.s

Re = G d / μ = 3435 ⇒ Φ = 0.005 (Figure 3.7)

K.E. = $G^2 \ln(P_1/P_2) = (0.41)^2 \ln(0.4/0.13)$ = 0.189 kg^2/(m^4.s^2)
Press.E. = $(P_2^2 - P_1^2) / (2\, P_1 \nu_1)$ = – 0.846 kg^2/(m^4.s^2)
Frc.E. = 4 Φ L/d G^2 = 0.6724 kg^2/(m^4.s^2)
[$(P_1 - P_2) / P_1$] % = 67.5%

Example -8.2-

A flow of 50 m^3/s methane, measured at 288 K and 101.3 kPa has to be delivered along a 0.6 m diameter line, 3km long a relative roughness e = 0.0001 m linking a compressor and a processing unit. The methane is to be discharged at the plant at 288 K and 170 kPa, and it leaves the compressor at 297 K. What pressure must be developed at the compressor in order to achieve this flow rate? Take that $\mu_{\text{CH4 at 293 K}} = 0.01 \times 10^{-3}$ Pa.s

Solution:

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2\,P_1 \upsilon_1} + 4\phi \frac{L}{d} G^2 = 0$$

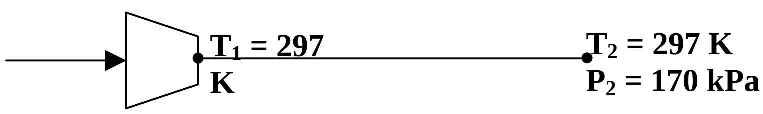
ΔT/L = 11ºC/3000 m = 0.00366ºC/m = 0.0366 ºC/10 m = 0.366 ºC/100 m = 3.66 ºC/1000 m

$$G = \frac{\dot{m}}{A} = \frac{Q\rho}{A} = \frac{Q}{A\upsilon} \qquad ,\upsilon = \frac{RT}{PMwt} = \frac{8.314\,(\text{Pa.m}^3/\text{mol.K})\ 288\text{K}\ [(10^3\ \text{mol/kmol})]}{(101.3\times 10^3\,\text{Pa})\quad 16\ \text{kg/kmol}} = 1.477 m^3/kg$$

$\Rightarrow \quad G = (50) / [(\pi/4\ 0.6^2)(1.477)] = 119.7$ kg/m^2.s

Since the difference in temperature is relatively small, therefore the processes could be consider isothermal at ($T = T_m$),

$T_m = (297 + 288)/2 = 293$ K



$$P_1\upsilon_1 = \frac{RT_m}{Mwt} = \frac{8314\,(\mathrm{Pa.m^3/kmol.K})\,293\mathrm{K}}{16\ \mathrm{kg/kmol}} = 1.5225\times10^5\ Pa.m^3/kg \quad or\left(J/kg \equiv m^2/s^2\right)$$

Re = G d / μ = 119.7(0.6)/0.01 x 10^{-3} = 7.182 x 10^6, e/d = 0.0001 / 0.6 = 0.00016

⇒ Φ = 0.0015 (Figure 3.7)

$$(119.7)^2 \ln\left(\frac{P_1}{170\times10^3}\right) + \frac{(170\times10^3 - P_1^2)}{2(1.5225\times10^5)} + 4(0.0015)\frac{3000}{0.6}(119.7)^2 = 0$$

⇒ $\ln P_1 - 2.292 \times 10^{-10} P_1^2 + 24.58 = 0 \quad \Rightarrow P_1 = \sqrt{\frac{\ln p_1 + 24.58}{2.292\times10^{-10}}}$

Solution by trial and error

P_1 Assumed	200 x 10^3	400.617 x 10^3	404.382 x 10^3	404.432 x 10^3
P_1 Calculated	400.617 x 10^3	404.382 x 10^3	404.432 x 10^3	404.433 x 10^3

⇒ P_1 = 404.433 x 10^3 Pa

K.E. = $G^2 \ln(P_1/P_2)$	**= 12418 $kg^2/(m^4.s^2)$**
Press.E.	**=– 442253 $kg^2/(m^4.s^2)$**
Frc.E.	**= 429842 $kg^2/(m^4.s^2)$**
[($P_1 - P_2$) / P_1] %	**= 58.5%**

Example -8.3-

Town gas, having a molecular weight 13 kg/kmol and a kinematic viscosity of 0.25 stoke is flowing through a pipe of 0.25 m I.D. and 5 km long at arate of 0.4 m^3/s and is delivered at atmospheric pressure. Calculate the pressure required to maintain this rate of flow. The volume of occupied by 1 kmol and 101.3 kPa may be taken as 24 m^3. What effect on the pressure required would result if the gas was delivered at a height of 150 m (i) above and (ii) below its point of entry into the pipe? e = 0.0005 m.

Solution:

$P_2 = P_1$ = 101.3 kPa

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

$$G = \frac{\dot m}{A} = \frac{Q\rho}{A} = \frac{Q}{A\upsilon} \qquad ,\upsilon = 24\frac{m^3}{kmol}\left(\frac{1}{13\ \mathrm{kg/kmol}}\right) = 1.846 m^3/kg$$

⇒ G = (0.4) / [(π/4 0.25^2)(1.846)] = 4.414 kg/m^2.s

Re = G d / μ = G d / (ρ ν) = 4.414 (0.25) / [(1/1.846) 0.25 x 10^{-4}] = 8.1489 x 10^4,

e/d = 0.0005 / 0.25 = 0.002 ⇒ Φ = 0.0031 (Figure 3.7)

As first approximation the kinetic energy term will be omitted

$$\frac{-\Delta P}{\upsilon_m} = \frac{(P_1 - P_2)}{\upsilon_m} = 4\phi\frac{L}{d}G^2, \qquad \upsilon_2 = 1.846\ m^3/kg\ ,\ \upsilon_1 = RT/(P_1\ Mwt)$$

⇒ υ_1 = (8314) (289) / [(P_1) 13] = 184.826 x $10^3/P_1$

$\upsilon_m = (\upsilon_1 + \upsilon_2)/2$ = [(184.826 x $10^3/P_1$) + 1.846]/2 = [92413.3+0.923 P_1]/ P_1

$$\Rightarrow \frac{(P_1 - 101.3\times10^3)P_1}{92413.3 + 0.923P_1} = 4(0.0031)\frac{5000}{0.25}(4.414)^2$$

$$\Rightarrow P_1^2 - 101.3\times10^3 P_1 = 4831.9(92413.3 + 0.923P_1) = 4.4653\times10^8 + 4459.8P_1$$