$\Rightarrow \quad P_1^2 - 105.76 \times 10^3 P_1 - 4.4653 \times 10^8 = 0$

$$x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

either $P_1 = 109.825 \times 10^3$ Pa

or $P_1 = -4065.8$ ------neglect

K.E. = $G^2 \ln(P_1/P_2)$ = 1.5744 kg^2/(m^4.s^2)

Press.E. =– 4831.9 kg^2/(m^4.s^2)

Frc.E. = 4831.9 kg^2/(m^4.s^2)

[$(P_1 - P_2) / P_1$] % = 7.7 %

∴ The first approximation is justified

If use the equation of the terms;

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1} + 4\phi \frac{L}{d} G^2 = 0 \text{------ Neglect the kinetic energy term}$$

$$\frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1} + 4\phi \frac{L}{d} G^2 = 0$$

$$P_1 \upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 289\text{K}}{13\ \text{kg/kmol}} = 184.8266 \times 10^3 \left(J/kg \equiv m^2/s^2\right)$$

$$\Rightarrow P_1^2 = P_2^2 + 2 P_1 \upsilon_1 \left(4\phi \frac{L}{d} G^2\right) = 0$$

$$= (101.3 \times 10^3)^2 + 2(184.8266 \times 10^3)\,[4(0.0031)(5000/0.25)(4.414)^2]$$

$$\Rightarrow P_1^2 = 1.20478 \times 10^{10} \Rightarrow P_1 = 109.762 \times 10^3 \text{ Pa}$$

If the pipe is not horizontal, the term (g dz) must be included in equation (**) or the term ($g\,\Delta z/\upsilon_m^2$) to integration of this equation [i.e. General equation of energy apply to compressible fluid in horizontal pipe with no shaft work]

$\upsilon_m = 1.7644$ m^3/kg, $\quad \upsilon_{air} = (8314 \times 289)/(101.3 \times 10^3 \times 29) = 0.8179$ m^3/kg

$\rho_m = 0.5668$ kg / m^3, $\quad \rho_{air} = 1.223$ kg / m^3

As gas is less dense than air, υ_m is replaced by ($\upsilon_{air} - \upsilon_m$) in potential energy term;

$$\frac{g\,\Delta z}{(\upsilon_{air} - \upsilon_m)^2} = \frac{9.81(150)}{(-0.9465)^2} = 1642.55 \quad kg^2/m^4.s^2 \qquad and \quad \frac{g\,\Delta z}{(\upsilon_{air} - \upsilon_m)} = 1555\ Pa$$

(i) Point ② 150 m above point ① ⇒ $P_1 = 109.762 \times 10^3 - 1555 = 108.207 \times 10^3$ Pa

(ii) Point ② 150 m below point ① ⇒ $P_1 = 109.762 \times 10^3 + 1555 = 111.317 \times 10^3$ Pa

Example -8.4-

Nitrogen at 12 MPa pressure fed through 25 mm diameter mild steel pipe to a synthetic ammonia plant at the rate of 1.25 kg/s. What will be the drop in pressure over a 30 m length of pipe for isothermal flow of the gas at 298 K? e = 0.0005 m, μ = 0.02 mPa.s

Solution:

$P_1 = 12$ MPa

First approximation [neglect the kinetic energy]

$$\Rightarrow \quad \frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1} + 4\phi \frac{L}{d} G^2 = 0$$

$$P_1 \upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 298\text{K}}{28\ \text{kg/kmol}} = 88484.7 \quad \left(J/kg \equiv m^2/s^2\right)$$

$$\Rightarrow P_2^2 = P_1^2 - 2P_1\upsilon_1\left(4\phi\frac{L}{d}G^2\right) = 0$$

$$G = \frac{\dot{m}}{A} = \frac{1.25}{\pi/4(0.025)^2} = 2546.48 \quad kg/m^2.s$$

Re = G d / μ = 2546.48 (0.025) / 0.02 x 10^{-3} = 3.183 x 10^6, e/d = 0.0002

⇒ Φ = 0.0017 (Figure 3.7)

$P_2^2 = (12 \times 10^6)^2 - 2(88484.7)\,[4(0.0017)(30/0.025)(2546.48)^2]$

⇒ $P_2 = 11.603 \times 10^6$ Pa

K.E. = $G^2 \ln(P_1/P_2)$	**= 2.1816 x 10^5 $kg^2/(m^4.s^2)$**
Press.E.	**= – 529.492 x 10^5 $kg^2/(m^4.s^2)$**
Frc.E.	**= 529.14 x 10^5 $kg^2/(m^4.s^2)$**
[($P_1 – P_2$) / P_1] %	**= 3.3 %**

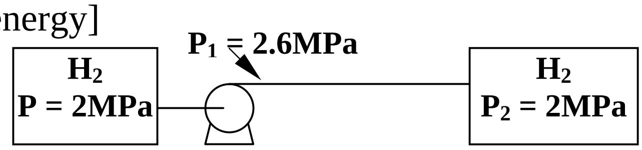
∴ the first approximation is justified

Example -8.5-

Hydrogen is pumped from a reservoir at 2 MPa pressure through a clean horizontal mild steel pipe 50 mm diameter and 500 m long. The downstream pressure is also 2 MPa. And the pressure of this gas is raised to 2.6 MPa by a pump at the upstream end of the pipe. The conditions of the flow are isothermal and the temperature of the gas is 293 K. What is the flow rate and what is the effective rate of working of the pump if η = 0.6 e = 0.05 mm, μ = 0.009 mPa.s.

Solution:

First approximation [neglect the kinetic energy]



$$\Rightarrow \quad \frac{(P_2^2 - P_1^2)}{2P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

$$P_1\upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\,293\text{K}}{2\ \text{kg/kmol}} = 121.8\times10^4 \quad (J/kg \equiv m^2/s^2)$$

P_1 = 2.6 MPa, P_2 = 2 MPa, $-\Delta P_f = P_1 - P_2 = 0.6 \times 10^6$ Pa

$\rho_m = 1/\upsilon_m = P_m\,Mwt/RT = (2.3 \times 10^6)\ 2 / (8314 \times 293) = 1.89\ kg/m^3$

$\Phi Re^2 = (-\Delta P_f/L)(\rho_m d^3/4\mu^2) = [(0.6\times10^6)/(500)][(1.89)(0.05)^3/(4)(0.009\times10^{-3})^2] = 8.75\times10^8$

e/d = 0.001 ⇒Figure (3.8) Re = 5.9 x10^5 ⇒ G = 5.9 x10^5 (0.009x10^{-3}) / (0.05)

⇒G = 106.2 kg / m^2.s

Re = 5.9 x10^5, e/d = 0.001 ⇒ Φ = 0.0025 (Figure 3.7)

$$\Rightarrow \quad G^2 = \frac{(P_2^2 - P_1^2)}{(2P_1\upsilon_1)(-4\phi L/d)} = \frac{(2\times10^6)^2 - (2.6\times10^6)^2}{(2\times121.8\times10^4)[-4(0.0025)(500/0.05)]} = 11330$$

⇒ G = 106.44 kg / m^2.s ------------------------∴ ok

K.E. = $G^2 \ln(P_1/P_2)$	**= 2.9726 x 10^3 $kg^2/(m^4.s^2)$**	∴ the neglecting the kinetic energy term is OK
Press.E.	**= – 1133.005 x 10^3 $kg^2/(m^4.s^2)$**	
Frc.E.	**= 1132.94736 x 10^3 $kg^2/(m^4.s^2)$**	
[($P_1 – P_2$) / P_1] %	**= 3.3 %**	

$$Power = \frac{\dot{m}P_1\upsilon_1 \ln(P_1/P_2)}{\eta} = \frac{0.209(121.8\times10^4)\ln(2.6/2)}{0.6} = 111.3\ kW$$

Example -8.6-

In the synthetic ammonia plant the hydrogen is fed through a 50 mm diameter steel pipe to the converters. The pressure drop over the 30 m length of pipe is 500 kPa, the pressure at the downstream end being 7.5 MPa. What power is required in order to overcome friction losses in the pipe? Assume isothermal expansion of the gas at 298 K. What error introduced by assuming the gas to be an incompressible fluid of density equal to that at the mean pressure in the pipe? μ = 0.02 mPa.s.

Solution:

P_2 = 7.5 MPa, $P_1 = P_2 + (-\Delta P_f)$ = 7.5 MPa + 0.5 MPa = 8MPa = 8 x 10^6 Pa

The pressure (P_m) = ($P_1 + P_2$)/2 = 7.75 x 10^6 Pa

[($P_1 - P_2$) / P_1] % = 6.25 %

$$\rho_m = \frac{P_m . Mwt}{RT} = \frac{7.75\times10^6(2)}{8314(298)} = 6.256 kg/m^3$$

For incompressible fluids

$$\frac{-\Delta P}{\rho_m} = 4\phi\frac{L}{d}u^2$$

$$\Rightarrow -\Delta P\,\rho_m = 4\phi\frac{L}{d}u^2\rho_m^2 = 4\phi\frac{L}{d}G^2 \quad \Rightarrow G^2 = \frac{-\Delta P\,\rho_m}{4\phi L/d}$$

Assume Φ = 0.003

$\Rightarrow$ G^2 = 434,444.444 $kg^2/m^4.s^2$ $\Rightarrow$ G = 659.124 $kg/m^2.s$

$\Rightarrow$ Re =1.647 x 10^6, and Φ = 0.003 $\Rightarrow$ from Figure (3.7) e/d = 0.00189

$\Rightarrow$ e = 0.09 mm (this value is reasonable for steel)

For compressible fluids

$$G^2\ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

$$G^2\ln\left(\frac{8}{7.5}\right) + \frac{(7.5\times10^6)^2 - (8\times10^6)^2}{2[8314(298)/2]} + 4(0.003)\frac{30}{0.05}G^2 = 0$$

$\Rightarrow$ G^2 = 430,593.418 $kg^2/m^4.s^2$ $\Rightarrow$ G = 656.2 $kg/m^2.s$

Very little error is made by the simplifying assumption in this particular case.

$$Power = \frac{\dot{m}P_1\upsilon_1\ln(P_1/P_2)}{\eta} = \frac{(656.2\times\pi/4\;0.005^2)(123.8786\times10^4)\ln(8/7.5)}{0.6} = 171.7\;kW$$

Example -8.7-

A vacuum distillation plant operating at 7 kPa pressure at top has a boil-up rate of 0.125 kg/s of xylene. Calculate the pressure drop along a 150 mm bore vapor pipe used to connect the column to the condenser. And also calculate the maximum flow rate if L = 6 m, e = 0.0003 m, Mwt = 106 kg/kmol, T = 338 K, μ = 0.01 mPa.s.

Solution:

$$G^2\ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

G = 0.125 / [π/4 $(0.15)^2$] = 7.074 $kg/m^2.s$

P_1 = 7 kPa, P_2 = Pressure at condenser