$$P_1\upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\,338\text{K}}{106\ \text{kg/kmol}} = 26510.68 \quad \left(J/kg \equiv m^2/s^2\right)$$

Re = G d / μ = 7.074(0.15)/0.01 x 10^{-3} = 1.06 x 10^5, e/d = 0.002

⇒ Φ = 0.003 (Figure 3.7)

$$\Rightarrow \ln\left(\frac{7\times10^3}{P_2}\right) + 3.769\times10^{-7}[P_2^2 - (7\times10^3)^2] + 4(0.003)\frac{6}{0.15} = 0$$

$$\Rightarrow \quad P_2^2 = \left(7\times10^3\right)^2 - \frac{\ln(7\times10^3/P_2)+0.48}{3.769\times10^{-7}} \quad \Rightarrow \quad P_2 = \sqrt{\left(7\times10^3\right)^2 - \frac{\ln(7\times10^3/P_2)+0.48}{3.769\times10^{-7}}}$$

Solution by trial and error

P_2 Assumed	5 x 10^3	6.8435 x 10^3	6.904 x 10^3	6.9057 x 10^3
P_2 Calculated	6.8435 x 10^3	6.904 x 10^3	6.9057 x 10^3	6.9058 x 10^3

⇒ P_2 = 6.9058 x 10^3 Pa

–ΔP = $P_1 - P_2$ = (7 – 6.9058) x 10^3 = 94.2Pa

[($P_1 - P_2$) / P_1] % = 0.665 % we can neglect the K.E. term in this problem

H.W. resolve this example with neglecting the K.E. term

For maximum flow rate calculations

$$\dot{m}_{max} = A\,P_w\sqrt{1/P_1\upsilon_1} \quad \Rightarrow \quad G_{max} = P_w\sqrt{1/P_1\upsilon_1}$$

To estimate P_w

$$\ln\left(\frac{P_1}{P_w}\right)^2 + 1 - \left(\frac{P_1}{P_w}\right)^2 + 8\phi\frac{L}{d} = 0$$

Let X ≡ $(P_1/P_w)^2$

⇒ ln(X) + 1 – X + 8 Φ L/d = 0 ⇒ X = 1.96 + ln(X)

Solution by trial and error

X Assumed	1.2	2.14	2.72	2.96	3.074	3.086	3.087
X Calculated	2.14	2.72	2.96	3.074	3.086	3.087	3.087

⇒ X = 3.087 = $(P_1/P_w)^2$ ⇒ $P_w = P_1/(3.087)^{0.5}$ = 3984 Pa

∴ the system does not reach maximum velocity (H.W. explain)

⇒ G_{max} = 3984 / $(26510.68)^{0.5}$ = 24.47 kg/m^2.s

Example -8.8-

A vacuum system is required to handle 10 g/s of vapor (molecular weight 56 kg/kmol) so as to maintain a pressure of 1.5 kN/m^2 in a vessel situated 30 m from the vacuum pump. If the pump is able to maintain a pressure of 0.15 kN/m^2 at its suction point, what diameter of pipe is required? The temperature is 290 K, and isothermal conditions may be assumed in the pipe, whose surface can be taken as smooth. The ideal gas law is followed. Gas viscosity μ = 0.01 mN s/m^2.

Solution:

$$G^2\ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

$$G = \frac{\dot{m}}{\pi/4\,d^2} = 10\times10^{-3}/[\pi/4\,(d)^2] = 0.0127\,d^{-2}$$

Re = G d / μ = 1273.25 d^{-1} ----------------------------------(1)

$$P_1\upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\;290\text{K}}{56\ \text{kg/kmol}} = 43054.64 \quad \left(J/kg \equiv m^2/s^2\right)$$

⇒ $2.3\,G^2 - 52.97 + 120\,\Phi/d\,G^2 = 0$

⇒ $3.733 \times 10^{-4}\,d^{-4} - 52.97 + 0.019\,d^{-3}\,\Phi = 0$

$$\Rightarrow \quad d = \left[\frac{52.97 - 0.019\,d^{-3}\phi}{3.733\times 10^{-4}}\right]^{-1/4}$$ ----------------------------------(2)

<u>Assume smooth pipe</u>

Solution by trial and error

		Eq.(1)		Figure (3.7)	Eq.(2)
Assume	d = 0.1 ⇒	Re = 1.3×10^{-4}	⇒	Φ = 0.0038 ⇒	d = 0.0515
	d = 0.0515 ⇒	Re = 2.5×10^{-4}	⇒	Φ = 0.0028 ⇒	d = 0.0516

∴ d = 0.0516 m.

8.3.2 Adiabatic Flow of an Ideal Gas in a Horizontal Pipe

The general energy equation of a steady-state flow system is: -

$dH + g\,dz + u\,du = dq - dW_s$

For adiabatic conditions ($dq = 0$) and in horizontal pipe ($dz = 0$) with no shaft work $dWs = 0$)

$\Rightarrow \quad dH + u\,du = 0$

but $G = \dfrac{\dot{m}}{A} = \rho\, u = \dfrac{u}{\upsilon} \quad \Rightarrow \quad u = \upsilon G$

$$\boxed{\begin{aligned} dH &= dU + d(P\upsilon) \\ c_p\,dT &= c_v\,dT + R\,dT \\ \therefore c_p &= c_v + R \end{aligned}}$$

$\Rightarrow \quad dH + G^2\,\upsilon\,d\upsilon = 0$

we have $\quad dH = c_p\,dT, \qquad$ and $dP\upsilon = RdT \quad \Rightarrow dT = dP\upsilon/R = dP\upsilon/(c_p - c_v)$

$\Rightarrow \quad dH = c_p\;[dP\upsilon/(c_p - c_v)] = (c_p / c_v)\,/[(c_p - c_v)/ c_v]\; dP\upsilon = [\gamma/(\gamma - 1)]\; dP\upsilon$

$$\boxed{\therefore \frac{\gamma}{\gamma-1}dP\upsilon + G^2\upsilon\, d\upsilon = 0}$$

The integration of this equation gives

$$\boxed{\frac{\gamma}{\gamma-1}P_1\upsilon_1 + \frac{G^2}{2}\upsilon_1^2 = \frac{\gamma}{\gamma-1}P_2\upsilon_2 + \frac{G^2}{2}\upsilon_2^2 = \frac{\gamma}{\gamma-1}P\upsilon + \frac{G^2}{2}\upsilon^2 = K}$$

This equation is used to estimate the downstream pressure P_2

To estimate the downstream specific volume υ_2 the procedure is as follow

$$\frac{\gamma}{\gamma-1}P\upsilon = K - \frac{G^2}{2}\upsilon^2 \Rightarrow P = \left(\frac{\gamma-1}{\gamma}\right)\left[\frac{K}{\upsilon} - \frac{G^2}{2}\upsilon\right]$$

$$\Rightarrow dP = \left(\frac{\gamma-1}{\gamma}\right)\left[-\frac{K}{\upsilon^2} - \frac{G^2}{2}\right]d\upsilon \qquad \div\,\upsilon$$

$$\Rightarrow \frac{dP}{\upsilon} = \left(\frac{\gamma-1}{\gamma}\right)\left[-\frac{K}{\upsilon^3} - \frac{G^2}{2\upsilon}\right]d\upsilon$$

$$\Rightarrow \int_{P_1}^{P_2}\frac{dP}{\upsilon} = \left(\frac{\gamma-1}{\gamma}\right)\left[\frac{K}{2}\left(\frac{1}{\upsilon_2^2} - \frac{1}{\upsilon_1^2}\right) - \frac{G^2}{2}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)\right]$$

But, $K = \dfrac{G^2}{2}\upsilon_1^2 + \dfrac{\gamma}{\gamma-1}P_1\upsilon_1$

$$\Rightarrow \int_{P_1}^{P_2}\frac{dP}{\upsilon} = \left(\frac{\gamma-1}{\gamma}\right)\left[\frac{G^2}{4}\upsilon_1^2\left(\frac{1}{\upsilon_2^2} - \frac{1}{\upsilon_1^2}\right) + \frac{\gamma}{\gamma-1}\frac{P_1\upsilon_1}{2}\left(\frac{1}{\upsilon_2^2} - \frac{1}{\upsilon_1^2}\right) - \frac{G^2}{2}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)\right]$$

$$= \frac{\gamma-1}{4\gamma}G^2\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2 - 1 - 2\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)\right] + \frac{P_1\upsilon_1}{2}\left(\frac{1}{\upsilon_2^2} - \frac{1}{\upsilon_1^2}\right)$$

But,

$$G^2\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) + \int_{P_1}^{P_2}\frac{dP}{\upsilon} + 4\phi\frac{L}{d}G^2 = 0$$

The general equation of energy apply to compressible fluid in horizontal pipe with no shaft work