$$\Rightarrow G^2 \ln\left(\frac{\upsilon_2}{\upsilon_1}\right) + \frac{\gamma-1}{4\gamma}G^2\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2 - 1 - 2\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)\right] + \frac{P_1\upsilon_1}{2}\left(\frac{1}{\upsilon_2^2} - \frac{1}{\upsilon_1^2}\right) + 4\phi\frac{L}{d}G^2 = 0 \cdots\cdots\cdots \times \frac{2}{G^2}$$

$$\Rightarrow 2\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) - \frac{\gamma-1}{\gamma}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) + \frac{\gamma-1}{2\gamma}\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2 - 1\right] + \frac{P_1}{\upsilon_1 G^2}\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2 - 1\right] + 8\phi\frac{L}{d} = 0$$

$$\Rightarrow \frac{2\gamma-\gamma+1}{\gamma}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) + \left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2 - 1\right]\left[\frac{\gamma-1}{2\gamma} + \frac{P_1}{\upsilon_1 G^2}\right] + 8\phi\frac{L}{d} = 0$$

$$\boxed{\Rightarrow \frac{\gamma+1}{\gamma}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) + \left[\frac{\gamma-1}{2\gamma} + \frac{P_1}{\upsilon_1 G^2}\right]\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2 - 1\right] + 8\phi\frac{L}{d} = 0}$$

This equation is used to estimate the downstream specific volume υ_2

8.3.2.1 Maximum Velocity in Adiabatic Flow

For constant upstream conditions, the maximum flow through the pipe is found by differentiating (G) with respect to (υ_2) of the last equation and putting ($dG/d\upsilon_2$) equal to zero.

The maximum flow is thus shown to occur when the velocity at downstream end of the pipe is the sonic velocity.

i.e. $\frac{dG}{d\upsilon_2} = 0 \quad \Rightarrow u_w = \sqrt{\gamma P_2 \upsilon_2} \quad \Rightarrow G_{max} = \frac{\sqrt{\gamma P_2 \upsilon_2}}{\upsilon_2} = \sqrt{\frac{\gamma P_2}{\upsilon_2}}$

Note: -

In isentropic (or adiabatic) flow [$P_1\upsilon_1 \neq P_2\upsilon_2$] where, in these conditions [$P_1\upsilon_1^{\gamma} \neq P_2\upsilon_2^{\gamma}$]

i.e. $u_w = \sqrt{\gamma P_2 \upsilon_2} \neq \sqrt{\gamma P_1 \upsilon_1}$

Typical values of (γ) for ordinary temperatures and pressures are: -

i- For monatomic gases such as He, Ar ($\gamma = 1.67$)

ii- For diatomic gases such as H_2, N_2, CO ($\gamma = 1.4$)

iii- For tritomic gases such as CO_2 ($\gamma = 1.3$)

Example -8.9-

Air, at a pressure of 10 MN/m^2 and a temperature of 290 K, flows from a reservoir through a mild steel pipe of 10 mm diameter and 30 m long into a second reservoir at a pressure P_2. Plot the mass rate of flow of the air as a function of the pressure P_2. Neglect any effects attributable to differences in level and assume an adiabatic expansion of the air. μ = 0.018 mN s/m^2, γ = 1.36.

Solution:

$$\frac{\gamma+1}{\gamma}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) + \left[\frac{\gamma-1}{2\gamma} + \frac{P_1}{\upsilon_1 G^2}\right]\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2 - 1\right] + 8\phi\frac{L}{d} = 0$$

$$\upsilon_1 = \frac{RT}{P_1\, Mwt} = \frac{8314\,(\mathrm{Pa.m^3/kmol.K})\ 290\mathrm{K}}{10\times10^6\,Pa\,(29\ \mathrm{kg/kmol})} = 8.314\times10^{-3}\,m^3/kg$$

$$\Rightarrow 1.735\ln\left(\frac{\upsilon_2}{8.314\times10^{-3}}\right) + \left[0.132 + \frac{1.2028\times10^9}{G^2}\right]\left[\left(\frac{8.314\times10^{-3}}{\upsilon_2}\right)^2 - 1\right] + 24000\phi = 0$$

$$\Rightarrow \left(\frac{8.314\times10^{-3}}{\upsilon_2}\right)^2 = 1-\frac{1.735\ln\left(\frac{\upsilon_2}{8.314\times10^{-3}}\right)+24000\phi}{0.132+\frac{1.2028\times10^9}{G^2}}$$

$$\Rightarrow \upsilon_2 = \frac{8.314\times10^{-3}}{\sqrt{1-\frac{1.735\ln\left(\frac{\upsilon_2}{8.314\times10^{-3}}\right)+24000\phi}{0.132+\frac{1.2028\times10^9}{G^2}}}} \text{ ----------------------------(1)}$$

$Re = Gd/\mu = 555.6\ G$ ----------------------------(2)

$$\frac{\gamma}{\gamma-1}P_1\upsilon_1+\frac{G^2}{2}\upsilon_1^2=\frac{\gamma}{\gamma-1}P_2\upsilon_2+\frac{G^2}{2}\upsilon_2^2 \qquad \Rightarrow \frac{\gamma}{\gamma-1}P_2\upsilon_2=\frac{\gamma}{\gamma-1}P_1\upsilon_1+\frac{G^2}{2}(\upsilon_1^2-\upsilon_2^2)$$

$$\Rightarrow P_2=\frac{P_1\upsilon_1}{\upsilon_2}+\frac{\gamma-1}{2\gamma}\frac{G^2}{\upsilon_2}(\upsilon_1^2-\upsilon_2^2)$$

$$\Rightarrow P_2=\frac{83140}{\upsilon_2}+0.132\frac{G^2}{\upsilon_2}(6.91\times10^{-5}-\upsilon_2^2) \text{ ----------------------------(3)}$$

1- at $P_2 = P_1 \Rightarrow G = 0$

eq.(2) Figure (3.7)

2- assume $G = 2000\ kg/m^2.s \Rightarrow Re = 1.11 \times 10^6 \Rightarrow \Phi = 0.0028$

Solution by trial and error

υ_2 Assumed	10×10^{-3}	9.44×10^{-3}
υ_2 Calculated eq.(1)	9.44×10^{-3}	9.44×10^{-3}

$\Rightarrow \quad \upsilon_2 = 9.44 \times 10^{-3}\ m^3/kg \quad \Rightarrow \quad P_2 = 8.8 \times 10^6\ Pa$

3- assume $G = 3000\ kg/m^2.s \Rightarrow Re = 1.6 \times 10^6 \Rightarrow \Phi = 0.0028$

Solution by trial and error

υ_2 Assumed	10×10^{-3}	11.8×10^{-3}
υ_2 Calculated eq.(1)	11.8×10^{-3}	11.81×10^{-3}

$\Rightarrow \quad \upsilon_2 = 11.84 \times 10^{-3}\ m^3/kg \quad \Rightarrow \quad P_2 = 7.013 \times 10^6\ Pa$

G ($kg/m^2.s$)	υ_2(m^3/kg)	P_2(Mpa)
0	8.314×10^{-3}	10
2000	9.44×10^{-3}	8.8
3000	11.81×10^{-3}	7.013
3500	16.5×10^{-3}	5.01
4000	25×10^{-3}	3.37
4238	39×10^{-3}	2.04

<u>Example -8.10-</u>

Nitrogen at 12 MN/m^2 pressure is fed through a 25 mm diameter mild steel pipe to a synthetic ammonia plant at the rate of 0.4 kg/s. **What** will be the drop in pressure over a 30 m length of pipe assuming <u>isothermal expansion</u> of the gas at 300 K? **What** is the average quantity of heat per unit area of pipe surface that must pass through the walls in order to maintain isothermal conditions? **What** would be the pressure drop in the pipe if it were <u>perfectly lagged</u>? **What** would be the maximum flow rate in each case? Or **what** would be the Mach number? μ = 0.02 mNs/m^2, γ = 1.36, e/d = 0.002.

<u>Solution:</u>

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1} + 4\phi \frac{L}{d} G^2 = 0$$

$$P_1\upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 300\text{K}}{28\ \text{kg/kmol}} = 89078.6 \quad \left(J/kg \equiv m^2/s^2\right)$$

$$G = \frac{\dot{m}}{\pi/4\, d^2} = 0.4 / [\pi/4\,(0.25)^2] = 814.9\ \text{kg/m}^2\,\text{.s}, \quad Re = G\,d/\mu = 1.02 \times 10^6$$

e/d = 0.002 $\Rightarrow$ Φ = 0.0028 Figure (3.7)

Neglect the K.E. term

$\Rightarrow$ $P_2^2 = P_1^2 - 2\,P_1\upsilon_1(4\,\Phi\,(L/d)\,G^2) = 1.4241 \times 10^{14}$

$\Rightarrow$ $P_2 = 11.93 \times 10^6$ Pa

K.E. = $G^2 \ln(P_1/P_2)$	**= 3.885×10^4 kg^2/(m^4.s^2)**	$\therefore$ the neglecting the kinetic energy term is OK
Press.E.	**= – 940.24×10^4 kg^2/(m^4.s^2)**	
Frc.E.	**= 892.5×10^4 kg^2/(m^4.s^2)**	
[($P_1 - P_2$) / P_1] %	**= 0.583 %**	

$\Rightarrow$ $-\Delta P = P_1 - P_2 = 0.07 \times 10^6$ Pa

P_1, υ_1, u_1 $\longrightarrow$ **Plant, $u_2 = 0$**

$$\underbrace{\cancel{dH}}_{\text{isothermal}} + g\,\underbrace{\cancel{dz}}_{\text{horizontal}} + u\,du = dq - \underbrace{\cancel{dW_s}}_{\text{no shaft work}}$$

$\Rightarrow$ $u\,du = dq$ $\Rightarrow$ $q = \Delta u^2/2 = u_1^2/2$ [since the veloity in the plant is taken as zero]

$\Rightarrow$ $q = (G\,\upsilon_1)^2/2 = [814.9(89078.6/12 \times 10^6)]^2 / 2 = 18.3$ J/kg

The total heat pass through the wall $q_T = \dot{m}\,q$ = 0.4 (18.3) = 7.32 W

Heat flux $q'' = q_T / A$ = q_T / (π d L) = 7.32 / [π (0.025) 30] = 3.1 W/m^2

It is clear that the heat flux is very low value that could be considered the process is adiabatic.

For adiabatic conditions

$$\frac{\gamma+1}{\gamma}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) + \left[\frac{\gamma-1}{2\gamma} + \frac{P_1}{\upsilon_1 G^2}\right]\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2 - 1\right] + 8\phi\frac{L}{d} = 0$$

$$\Rightarrow \upsilon_2 = \frac{\upsilon_1}{\sqrt{1 - \dfrac{\dfrac{\gamma+1}{\gamma}\ln\left(\dfrac{\upsilon_2}{\upsilon_1}\right) + 8\phi\dfrac{L}{d}}{\dfrac{\gamma-1}{2\gamma} + \dfrac{P_1}{G^2\upsilon_1}}}} = \frac{7.423\times10^{-3}}{\sqrt{1 - \dfrac{1.714\ln\left(\dfrac{\upsilon_2}{7.423\times10^{-3}}\right) + 26.88}{0.143 + 2434.336}}}$$