Example -8.11-

Air enters at a pressure of 3.5 MPa and a temperature of 500ºC. The air flow rate through the nozzle is 1.3 kg/s and it leaves the nozzle at a pressure of 0.7 MPa. The expansion of air may be considered adiabatic. Calculate the area of throat and the exit area. Take $\gamma = 1.4$.

Solution:

$$A_2^2 = \dot{m}^2 \frac{(\gamma-1)}{2\gamma}\left(\frac{\upsilon_1}{P_1}\right)\left[\frac{(P_2/P_1)^{-\frac{2}{\gamma}}}{\left\{1-(P_2/P_1)^{\frac{\gamma-1}{\gamma}}\right\}}\right]$$

$$\upsilon_1 = \frac{RT_1}{P_1 Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\;773.15\text{K}}{3.5\times10^6\,Pa(29\text{ kg/kmol})} = 0.0633 \quad m^3/kg$$

$r = P_2/P_1$, $\quad r_c = P_{critical}/P_1 = P_{critical}/P_1 \quad \Rightarrow \quad r_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528$

$\Rightarrow P_{critical} = P_t = 0.528\ (3.5\text{ MPa}) = 1.85\text{ MPa}$

but $P_2 = 0.7$ MPa $\quad$ [i.e. $P_2 < P_t$] $\Rightarrow$ The case is (**III**)

at throat

$$A_t^2 = (1.3)^2\frac{0.4}{2.8}\left(\frac{0.0633}{3.5\times10^6}\right)\left[\frac{(0.528)^{-\frac{2}{1.4}}}{\left\{1-(0.528)^{\frac{0.4}{1.4}}\right\}}\right] \qquad \Rightarrow A_t = 2.55\times10^{-4}\,m^2$$

$\Rightarrow$ the diameter of throat $d_t = 18$ mm

At exit $(P_2/P_1) = 0.7/3.5 = 0.2$

$$\Rightarrow A_t^2 = (1.3)^2\frac{0.4}{2.8}\left(\frac{0.0633}{3.5\times10^6}\right)\left[\frac{(0.2)^{-\frac{2}{1.4}}}{\left\{1-(0.2)^{\frac{0.4}{1.4}}\right\}}\right] \qquad \Rightarrow A_t = 3.436\times10^{-4}\,m^2$$

$\Rightarrow$ the diameter of exit region $d_E = 21$ mm

Or another method

$u_t = u_w = \sqrt{\gamma P_t \upsilon_t}$ $\quad P_t = 1.85$ MPa $\qquad \upsilon_t = \upsilon_1 (P_t/P_1)^{-1/\gamma} = 0.0633\ (0.528)^{-1/1.4} = 0.0999 m^3/kg$

$\Rightarrow u_t = \sqrt{1.4(1.85\times10^6)(0.0999)} = 508.666 m/s$ (Sonic velocity)

$$A_t = \dot{m}\frac{\upsilon_t}{u_t} = 1.3\ (0.0999/508.666) = 2.55\times10^{-4}\,m^2$$

Or another method

$$u_2^2 = \left(\frac{2\gamma}{\gamma-1}\right)P_1\upsilon_1\left[1-(P_2/P_1)^{\frac{\gamma-1}{\gamma}}\right] = 1{,}550{,}850\left[1-(P_2/P_1)^{\,0.2857}\right]$$

$u_t^2 = 258671.997 \Rightarrow u_t = 508.6$ m/s

$u_2^2 = 571666.52 \Rightarrow u_2 = 756.086$ m/s

$\upsilon_2 = \upsilon_1 (P_2/P_1)^{-1/\gamma} = 0.0633(0.2)^{-1/1.4} = 0.1998\ m^3/kg$

$A_2 = \dot{m}\,\upsilon_2/u_2 = 1.3\ (0.198/756.086) = 3.436\times10^{-4}\,m^2$

8.5 Flow Measurement for Compressible Fluid

For horizontal flow with no shaft work and neglecting the frictional energy tem, the net of the general energy will be: -

$$\frac{u_2^2}{2\alpha_2} - \frac{u_1^2}{2\alpha_1} + \int_{P_1}^{P_2} \upsilon dP = 0$$

but $\dot{m}_1 = \dot{m}_2 = \dot{m} \quad \Rightarrow \quad u_1 = \frac{\upsilon_1}{\upsilon_2}\frac{A_2}{A_1}u_2$

☼ For isothermal flow

$$\int_{P_1}^{P_2} \upsilon dP = P_1\upsilon_1 \ln\frac{P_2}{P_1}$$

$$\Rightarrow u_2^2 - \left(\frac{\upsilon_1}{\upsilon_2}\frac{A_2}{A_1}u_2\right)^2 \frac{\alpha_2}{\alpha_1} + 2\alpha_2 P_1\upsilon_1 \ln\frac{P_2}{P_1} = 0$$

$$\boxed{\Rightarrow u_2^2 = \frac{2\alpha_2 P_1\upsilon_1 \ln(P_2/P_1)}{1 - \frac{\alpha_2}{\alpha_1}\left(\frac{\upsilon_1}{\upsilon_2}\frac{A_2}{A_1}\right)^2}} \qquad \text{-------------------------------(1)}$$

☼ For adiabatic flow

$$\upsilon = \upsilon_1 P_1^{\frac{1}{\gamma}} P^{-\frac{1}{\gamma}}$$

$$\int_{P_1}^{P_2} \upsilon dP = \upsilon_1 P_1^{\frac{1}{\gamma}} \int_{P_1}^{P_2} P^{-\frac{1}{\gamma}} dP = \frac{\gamma}{\gamma-1}\upsilon_1 P_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\boxed{\Rightarrow u_2^2 = \frac{2\alpha_2 P_1\upsilon_1\left(\frac{\gamma}{\gamma-1}\right)\left[(P_2/P_1)^{\frac{\gamma-1}{\gamma}} - 1\right]}{1 - \frac{\alpha_2}{\alpha_1}\left(\frac{\upsilon_1}{\upsilon_2}\frac{A_2}{A_1}\right)^2}} \qquad \text{-------------------------------(2)}$$

It should be noted that equations (1) and (2) apply provided that (P_2/P_1) is greater than the critical pressure ratio (r_c). Where if $(P_2/P_1) < (r_c)$, the flow becomes independent on P_2 and conditions of maximum flow occur.

8.6 Fans, Blowers, and Compression Equipment

Fans and blowers are used for many types of ventilating work such as air-conditioning systems. In large buildings, blowers are often used due to the high delivery pressure needed to overcome the pressure drop in the ventilation system.

Blowers are also used to supply draft air to boilers and furnaces.

Fans are used to move large volumes of air or gas through ducts, supplying air to drying, conveying material suspended in the gas stream, removing fumes, condensing towers and other high flow, low pressure applications.

Fans are used for low pressure where generally the delivery pressure is less than 3.447 kPa (0.5 psi), and blowers are used for higher pressures. However they are usually below delivery pressure of 10.32 kPa (1.5 psi). These units can either be **centrifugal** or the **axial-flow** type.

The axial flow type in which the air or gas enters in an axial direction and leaves in an axial direction.

The centrifugal blowers in which the air or gas enters in the axial direction and being discharge in the radial direction.

Compressors

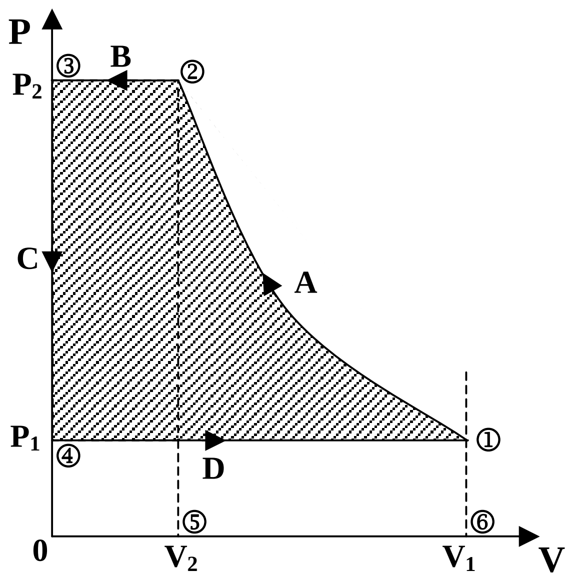
Compressor are used to handle large volume of gas at pressures increase from 10.32 kPa (1.5 psi) to several hundred kPa or (psi). Compressors are classified into: -

1- Cotinuous-flow compressors
   1-a- Centrifugal compressors
   1-b- Axial-flow compressors
2- Positive displacement compressors
   2-a- Rotary compressors
   2-b- Reciprocating compressors

Since a large proportion of the energy of compression appears as heat in the gas, there will normally be a considerable increase in temperature, which may limit the operation of the compressors unless suitable cooling can be effected. For this reason gas compression is often carried out in a number of stages and the gas is cooled between each stage.

8.7 Gas Compression Cycle

Suppose that, after the compression of a volume V_1 of gas at P_1 to a pressure P_2, the whole of the gas is expelled at constant pressure P_2, and a fresh charge of gas is admitted at a pressure P_1. The cycle can be followed as in Figure, where **P** is plotted as **ordinate** against **V** as **abscissa**.



Point ① represents the initial conditions of the gas of pressure and volume of (P_1,V_1).

► A-line ①→② Compression of gas from (P_1,V_1) to (P_2,V_2).

► B-line ②→③ Expulsion of gas at constant pressure P_2.