► C-line ③→④ Sudden reduction in pressure in the cylinder from P_2 to P_1. As the whole of the gas has been expelled.

► D-line ④→① A fresh charge of the gas through the suction stroke of the piston, during which a volume V_1 of gas is admitted at constant pressure P_1.

<u>The Total Work Done Per Cycle</u>

It will be noted that the mass of gas in the cylinder varies during the cycle. The work done by the compressor during each of the cycle is as follows: -

- Step (A): Compression $\quad -\int_{V_1}^{V_2} PdV \quad$ [area ①→②→⑤→⑥]
- Step (B): Expulsion $\quad P_2V_2 \quad$ [area ②→③→**0**→⑤]
- Step (D): Suction $\quad -P_1V_1 \quad$ [area ④→**0**→⑥→①]

∴ the total work done per cycle $= -\int_{V_1}^{V_2} PdV + P_2V_2 - P_1V_1$

= [area ①→②→③→④]

$$dPV = P\,dV + V\,dP \quad \Rightarrow \quad P\,dV = dPV - V\,dP$$

$$-\int_{V_1}^{V_2} PdV = \int_{P_1}^{P_2} VdP - \int_{P_1V_1}^{P_2V_2} dPV$$

but $PV = RT$ and $dPV = R\,dT$

$$\Rightarrow \int_{P_1V_1}^{P_2V_2} dPV = R\int_{T_1}^{T_2} dT = RT_2 - RT_1 = P_2V_2 - P_1V_1$$

$$\Rightarrow -\int_{V_1}^{V_2} PdV = \int_{P_1}^{P_2} VdP - (P_2V_2 - P_1V_1)$$

⇒ the total work done per cycle $= \int_{P_1}^{P_2} VdP - \cancel{P_2V_2} + \cancel{P_1V_1} + \cancel{P_2V_2} - \cancel{P_1V_1}$

$$= \int_{P_1}^{P_2} VdP$$

<u>Or</u> The total work done per cycle (W) $= -\int_{V_1}^{V_2} PdV + \Delta PV$

$$\Rightarrow \quad dW = -P\,dV + dPV \quad = -\cancel{PdV} + VdP + \cancel{PdV}$$

$$\Rightarrow \quad dW = dPV \quad \Rightarrow \quad W = \int_{P_1}^{P_2} VdP$$

Under isothermal conditions

The work of compression for an ideal gas per cycle $= \int_{P_1}^{P_2} VdP = RT\int_{P_1}^{P_2} dP/P$

$= RT\ln(P_2/P_1) = P_1V_1\ln(P_2/P_1)$

Under adiabatic conditions

The work of compression for an ideal gas per cycle $= \int_{P_1}^{P_2} VdP = V_1 P_1^{1/\gamma}\int_{P_1}^{P_2} P^{-1/\gamma} dP$

$= P_1V_1\,\gamma/(\gamma - 1)\,[(P_2/P_1)^{(\gamma-1)/\gamma} - 1]$

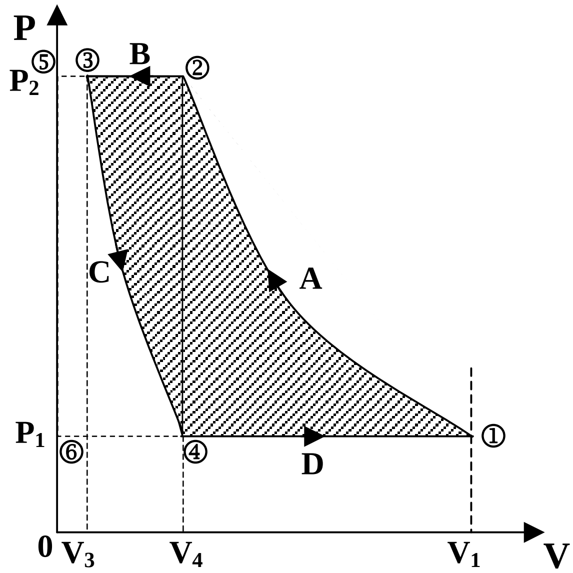
8.7.1 <u>Clearance Volume</u>

It practice, it is not possible to expel the whole of the gas fro the cylinder at the end of the compression; the volume remaining in the cylinder after the forward stroke of the piston is termed "**the clearance volume**".

The volume displaced by the piston is termed "**the swept volume**", and therefore **<u>the total volume of the cylinder</u>** is made up of <u>the clearance volume</u> plus <u>the swept volume</u>.

i.e. Total volume of cylinder = [clearance volume + swept volume]

A typical cycle for a compressor with a finite clearance volume can be followed by reference to the Figure;



A volume V_1 of gas at a pressure P_1 is admitted to the cylinder; its condition is represented by point ①**,**

► A-line ①→② Compression of gas from (P_1,V_1) to (P_2,V_2).

► B-line ②→③ Expulsion of gas at constant pressure P_2, so that the volume remaining in the cylinder is V_3.

► C-line ③→④ Expansion of this residual gas to the lower pressure P_1 and volume V_4 during the return stroke.

► D-line ④→① Introduction of fresh gas into the cylinder at constant pressure P_1.

<u>The Total Work Done Per Cycle</u>

The work done by the compressor during each of the actual cycle is as follows: -

- Step (A): Compression $\qquad -\int_{V_1}^{V_2} \boldsymbol{PdV}$
- Step (B): Expulsion $\qquad P_2(V_2-V_3)$
- Step (C): Expansion $\qquad -\int_{V_3}^{V_4} \boldsymbol{PdV}$
- Step (D): Suction $\qquad -P_1(V_1-V_4)$

The total work done per cycle is equal to the sum of these four components. It is represented by the selected area [i.e. area ①→②→③→④**]**, which is equal to [area①→②→⑤→⑥**]** less [area ③→④→⑤→⑥**]**

Under isentropic conditions

The work done per cycle

$$= \int_{P_1}^{P_2} \boldsymbol{VdP} - \int_{P_4}^{P_3} \boldsymbol{VdP}$$

$$= \frac{\gamma}{\gamma-1} P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] - \frac{\gamma}{\gamma-1} P_4V_4\left[\left(\frac{P_3}{P_4}\right)^{(\gamma-1)/\gamma} - 1\right]$$

but $(P_1 = P_4)$ and $(P_2 = P_3)$ ⇒

$$\boxed{W = \frac{\gamma}{\gamma-1} P_1(V_1 - V_4)\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]}$$

Now V_4 os not know explicitly, but can be calculated in terms of V_3, the clearance volume, for isentropic conditions

$$V_4 = V_3\,(P_2/P_1)^{1/\gamma}$$

And $\quad V_1 - V_4 = (V_1 - V_3) + V_3 - V_3(P_2/P_1)^{1/\gamma}$

$$= (V_1 - V_3)\,[1 + \{V_3/(V_1 - V_3)\} - \{V_3/(V_1 - V_3)\}\,(P_2/P_1)^{1/\gamma}]$$

where

$(V_1 - V_3) = V_s$: the swept volume

V_3 : the clearance volume

$V_3/(V_1 - V_3) = C$: the clearance

$\Rightarrow \quad V_1 - V_4 = V_s\,[1 + C - C\,(P_2/P_1)^{1/\gamma}]$

$\therefore$ The total work done on the fluid per cycle is therefore,

$$\boxed{\boldsymbol{W = \frac{\gamma}{\gamma - 1} P_1 V_s \left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{1/\gamma}\right]}}$$

The factor $\left[\boldsymbol{1 + C - C\left(\frac{P_2}{P_1}\right)^{1/\gamma}}\right]$ is called "**the theoretical volumetric efficiency**", and is a measure of the effect of the clearance on an isentropic compression.

The as is frequently **cooled** during compression so that the work done per cycle is less than that given by the last equation, (γ) is replaced by some smaller quantity (k). the greater the rate of heat removal, the less is the work done.

Notice that the isothermal compression is usually taken as the condition for the least work of compression. The actual work of compression is greater than the theoretical work because of clearance gases, back leakage, and frictional effects, where,

$$\eta = W_{theo}/W_{act}$$

8.8 Multistage Compressors

The maximum pressure ratio normally obtained in a single cylinder is (10) but values above (6) are usual. If the required pressure ratio (P_2/P_1) is large, it is not practicable to carry out the whole of the compression in a single cylinder because of the high temperatures, which would be set up, and the adverse effects of clearance volume on the efficiency. Further, lubrication would be difficult due to carbonization of the oil and there would be a risk of causing oil mist explosions in the cylinders when gases containing oxygen were being compressed.

The operation of the multistage compressor can conveniently be followed again on a pressure-volume diagram as shown in the Figure,

The [area ①→②→③→④**]** represents the work done in compressing isentropically from P_1 to P_2 in a single stage. The [area①→②→⑤→④⑥**]** represents the necessary work for an isothermal compression.

Now consider a multistage isentropic compression in which the intermediate pressures are P_{i1}, P_{i2}, P_{i3}, ………..etc.

The gas will be assumed to be cooled to its initial temperature in an inter-stage cooler before it enters each cylinder.