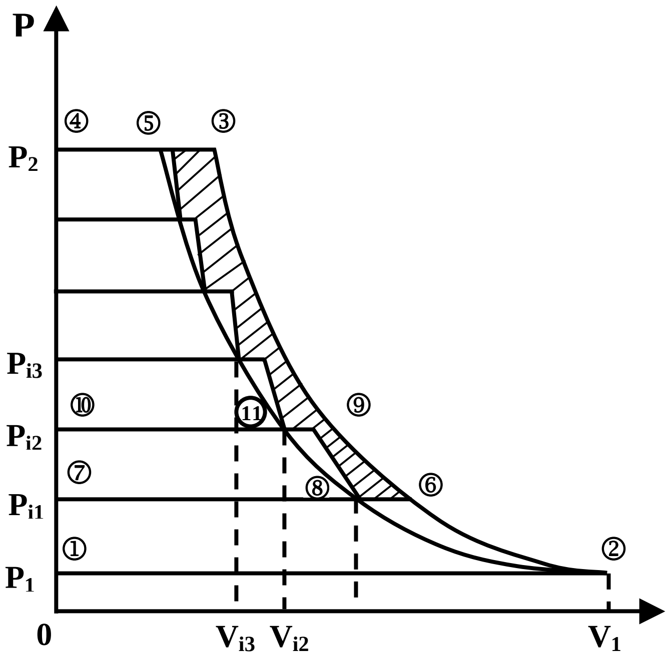
- ► A-line ①→② represents the suction stroke of the first stage where a volume (V_1) of gas is admitted at a pressure (P_1).
- ► B-line ②→⑥ represents an isentropic compression to a pressure (P_{i1}).
- ► C-line ⑥→⑦ represents the delivery of the gas from the first stage at a constant pressure (P_{i1}).
- ► D-line ⑦→⑧ represents the suction stroke of the second stage. The volume of the gas has the reduced in the inter-stage cooler to (V_{i1}), that which would have been obtained as a result of an isothermal compression to (P_{i2}).



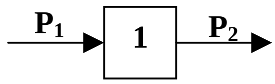
- ► E-line ⑧→⑨ represents an isentropic compression in the second stage from a pressure (P_{i1}) to a pressure (P_{i2}).
- ► F-line ⑨→⑩ represents the delivery stroke of the second stage.
- ► G-line ⑩→⑪ represents the suction stroke of the third stage point ⑪ again lyses on the line ②→⑤ that representing an isothermal compression.

It is seen that the overall work done on the gas is intermediate between that for a single stage isothermal compression and that for isentropic compression. The net saving in energy is shown as the shaded area in the last Figure.

<u>The Total Work Done for Multistage Compressors</u>

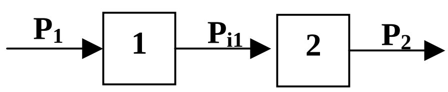
The total work done for compression the gas from P_1 to P_2 in an ideal single stage is,

$$W = \frac{\gamma}{\gamma - 1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right]$$



The total work done for compression the gas from P_1 to P_2 in an ideal two stages is,

$$W = \frac{\gamma}{\gamma - 1} P_1 V_1 \left[\left(\frac{P_{i1}}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right] + \frac{\gamma}{\gamma - 1} P_{i1} V_{i1} \left[\left(\frac{P_2}{P_{i1}} \right)^{(\gamma-1)/\gamma} - 1 \right]$$

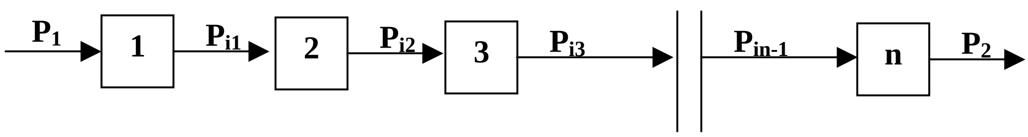


but for perfect inter-stage cooling i.e. at isothermal line $P_1V_1 = P_{i1}V_{i1}$= constant

$$\Rightarrow W = \frac{\gamma}{\gamma - 1} P_1 V_1 \left[\left\{ \left(\frac{P_{i1}}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right\} + \left\{ \left(\frac{P_2}{P_{i1}} \right)^{(\gamma-1)/\gamma} - 1 \right\} \right]$$

The total work done for compression the gas from P_1 to P_2 in an ideal n-stages is,

$$W = \frac{\gamma}{\gamma - 1} P_1 V_1 \left[\left(\frac{P_{i1}}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right] + \frac{\gamma}{\gamma - 1} P_{i1} V_{i1} \left[\left(\frac{P_{i2}}{P_{i1}} \right)^{(\gamma-1)/\gamma} - 1 \right] + \cdots\cdots\cdots + \frac{\gamma}{\gamma - 1} P_{in-1} V_{in-1} \left[\left(\frac{P_2}{P_{in-1}} \right)^{(\gamma-1)/\gamma} - 1 \right]$$



for perfect inter-stage cooling $P_1V_1 = P_{i1}V_{i1} = P_{i2}V_{i2} =$ ------ $= P_{in-1}V_{in-1} =$ constant

$$W = \frac{\gamma}{\gamma-1} P_1V_1\left[\left\{\left(\frac{P_{i1}}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right\} + \left\{\left(\frac{P_{i2}}{P_{i1}}\right)^{(\gamma-1)/\gamma} - 1\right\} + \cdots\cdots + \left\{\left(\frac{P_2}{P_{in-1}}\right)^{(\gamma-1)/\gamma} - 1\right\}\right]$$

$$\Rightarrow W = \frac{\gamma}{\gamma-1} P_1V_1\left[\left(\frac{P_{i1}}{P_1}\right)^{(\gamma-1)/\gamma} + \left(\frac{P_{i2}}{P_{i1}}\right)^{(\gamma-1)/\gamma} + \cdots\cdots + \left(\frac{P_2}{P_{in-1}}\right)^{(\gamma-1)/\gamma} - n\right]$$

The optimum values of intermediate pressures P_{i1}, P_{i2}, P_{i3}, -------P_{in-1} are so that **<u>the compression ratio (r) is the same in each stage and equal work is then done in each stage.</u>**

i.e. $\frac{P_{i1}}{P_1} = \frac{P_{i2}}{P_{i1}} = \frac{P_{i3}}{P_{i2}} = \cdots\cdots\cdots = \frac{P_2}{P_{in-1}} = r$

then $\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = \frac{P_{i1}}{P_1} = \frac{P_{i2}}{P_{i1}} = \frac{P_{i3}}{P_{i2}} = \cdots\cdots\cdots = \frac{P_2}{P_{in-1}} = r$ -------------prove that

$$\Rightarrow W = \frac{\gamma}{\gamma-1} P_1V_1\left[n\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - n\right] \qquad \Rightarrow \boxed{W = P_1V_1\frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right]}$$

The effect of clearance volume can now be taken into account. If the clearance in the successive cylinders are C_1, C_2 C_3, -------C_n the theoretical volumetric efficiency of the first cylinder = $\left[1 + C_1 - C_1(P_{i1}/P_1)^{1/\gamma}\right]$.

Assuming that the same compression ratio is used in each cylinder, then the theoretical volumetric efficiency of the first stage = $\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right]$.

If the swept volumes of the cylinders are V_{s1}, V_{s2}, V_{s3}, ----------- the volume of the gas admitted to the first cylinder = $V_{s1}\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right]$

The same mass of gas passes through each of the cylinders and, therefore, if the inter-stage coolers are assumed perfectly efficient, the ratio of the volumes of gas admitted to successive cylinder is $(P_1/P_2)^{1/n}$ [because lies on the isothermal line].

The volume of gas admitted to the second cylinder

$$= V_{s2}\left[1 + C_2 - C_2(P_2/P_1)^{1/n\gamma}\right] = V_{s1}\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right](P_1/P_2)^{1/n}$$

$$\boxed{\Rightarrow \qquad \frac{V_{s1}}{V_{s2}} \frac{\left[1 + C_2 - C_2(P_2/P_1)^{1/n\gamma}\right]}{\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right]}(P_2/P_1)^{1/n}}$$

In this manner the swept volume of each cylinder can be calculated in terms of V_{s1}, and C_1, C_2, …………., and the cylinder dimensions determined.

Let $V_1 = V_{s1}\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right]$, $\qquad V_2 = V_{s2}\left[1 + C_2 - C_2(P_2/P_1)^{1/n\gamma}\right]$

Where, V_i : represents the volume of gas admitted to stage i.

But for perfectly cooled [i.e. isothermal] $\Rightarrow P_1V_1 = P_{i1}V_1 = P_{i2}V_2 =$ --------$= P_{in-1}V_n$

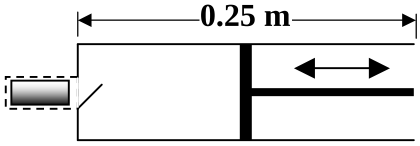
$$\Rightarrow P_1V_{s1}\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right] = P_{i1}V_{s2}\left[1 + C_2 - C_2(P_2/P_1)^{1/n\gamma}\right]$$

$$\textit{but} \quad r = \frac{P_{i1}}{P_1} = \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$\Rightarrow \frac{V_{s1}}{V_{s2}} = \frac{\left[1 + C_2 - C_2(P_2/P_1)^{1/n\gamma}\right]}{\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right]}\left(\frac{P_2}{P_1}\right)^{1/n}$$

Example -8.12-

A single-acting air compressor supplies 0.1 m^3/s of air (at STP) compressed to 380 kPa from 101.3 kPa. If the suction temperature is 289 K, the stroke is 0.25 m, and the speed is 4 Hz, what is the cylinder diameter? Assume the cylinder clearance is 4% and compression and re-expansion are isentropic (γ=1.4). What are the theoretical power requirements for the compression?



Solution:

Stroke ("حركة من سلسلة حركات متسلسلة "متوالية ومتشابهة)

Volume of gas per stroke = $(0.1\ m^3/s)/4s^{-1}\ (289/273)$

= $0.0264\ m^3$

=$(V_1–V_4)$ ≡[volume of gas admitted per cycle]

$P_2/P_1 = 380/101.3 = 3.75$

$(V_1–V_4) = V_s\ [1+ C – C\ (P_2/P_1)^{1/\gamma}]$

$0.0264 = V_s[1+0.04-0.04(3.75)^{1/1.4} \Rightarrow V_s = 0.0283\ m^3 = (V_1 – V_3) \equiv$ volume of cylinder

Cross-section area of cylinder = $V_s/L_{stroke} = 0.0283/0.25 = 0.113\ m^2$

$\Rightarrow$ The diameter of cylinder = $[0.113/(\pi/4)]^{1/2}$ =0.38 m

$$W = \frac{\gamma}{\gamma - 1} P_1(V_1 - V_4)\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$$

for 1kg of gas that compressed [or per cycle]

$$\Rightarrow W = \frac{1.4}{0.4}(101.3\times10^3)(0.0264)\left[(3.75)^{0.4/1.4} - 1\right] = 4278\ J/kg \text{ per stroke}$$

The theoretical power required = 4278 J/kg ($4s^{-1}$) per stroke= 17110 W =17.11kW

Example -8.13-

Air at 290 K is compressed from 101.3 kPa to 2065 kPa in two-stage compressor operating with a mechanical efficiency of 85%. The relation between pressure and volume during the compression stroke and expansion of clearance gas is ($PV^{1.25}$ = constant). The compression ratio in each of the two cylinders is the same, and the inter-stage cooler may be assumed 100% efficient. If the clearance in the two cylinders are 4% and 5%, calculate:

a- The work of compression per kg of air compressed;
b- The isothermal efficiency;
c- The isentropic efficiency;
d- The ratio of swept volumes in the two cylinders.

Solution:

$P_2/P_1 = 2065/101.3 = 20.4$

$$V_1 = \frac{RT}{P_1 Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 290\text{K}}{(101.3\times10^3\ Pa)29\ \text{kg/kmol}} = 0.82 \quad (m^3/kg)$$

For 100% efficient of cooler at inter-stage, the work of compression in multistage compressor of n-stages is;

$$W = P_1V_1\frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right] \Rightarrow W = (101.3\times10^3\times0.82)\frac{2(1.25)}{1.25-1}\left[(20.4)^{(0.25)/2.5} - 1\right] = 292.35\frac{kJ}{kg}$$