The work of compressor = W_{act} = W/η =292.3/0.85 = 344 kJ/kg
For isothermal compression = $W_{iso} = P_1V_1 \ln(P_2/P_1)$ = 250.5 kJ/kg
Isothermal efficiency = (W_{iso}/ W_{act}) 100 = 72.8 %
For isentropic compression = $W_{adb} = P_1V_1\, \gamma/(\gamma - 1)\, [(P_2/P_1)^{(\gamma-1)/\gamma} - 1]$ = 397.4 kJ/kg
Isentropic efficiency = (W_{adb}/ W_{act}) 100 = 115.5 %

$$V_1 = V_{s1}\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right]$$

$$\Rightarrow 0.82 = V_{s1}\left[1 + 0.04 - 0.04(20.4)^{1/2.5}\right] \Rightarrow V_{s1} = 0.905\ m^3/kg$$

The swept volume of the second cylinder is given by:

$$V_{s2} = V_{s1}\frac{\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right]}{\left[1 + C_2 - C_2(P_2/P_1)^{1/n\gamma}\right]}\left(\frac{P_1}{P_2}\right)^{1/n} \longleftarrow \text{انتبه}$$

$$V_{s2} = \frac{V_1(P_1/P_2)^{1/n}}{\left[1 + C_2 - C_2(P_2/P_1)^{1/n\gamma}\right]} = \frac{0.82(1/20.4)^{1/2}}{\left[1 + 0.05 - 0.05(20.4)^{1/2.5}\right]} = 0.206\ m^3/kg$$

∴ V_{s1}/V_{s2} = 0.905/0.206 = 4.4

<u>Example -8.14-</u>

Calculate the theoretical work in (J/kg) required to compress a diatomic gas initially at T = 200 K adiabatically compressed from a pressure of 10 kPa to 100 kPa in;

1- Single stage compressor;
2- Two equal stages;
3- Three equal stages; Taken that γ = 1.4, Mwt = 28 kg/kmol

<u>Solution:</u>

1- $$W = P_1V_1\frac{\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$$

P_2/P_1 = 100/10 = 10

$$V_1 = \frac{RT}{P_1 Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 200\text{K}}{(10 \times 10^3\ Pa)28\ \text{kg/kmol}} = 5.94\quad (m^3/kg)$$

$$\Rightarrow W = 10(5.92)\frac{1.4}{0.4}\left[(10)^{0.4/1.4} - 1\right] = 193.44\ kJ/kg$$

2- $$W = P_1V_1\frac{n\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right] \Rightarrow W = 59.4\frac{2(1.4)}{0.4}\left[(10)^{(0.4)/2.8} - 1\right] = 161.95\frac{kJ}{kg}$$

3- $$W = P_1V_1\frac{n\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right] \Rightarrow W = 59.4\frac{3(1.4)}{0.4}\left[(10)^{(0.4)/4.2} - 1\right] = 152.93\frac{kJ}{kg}$$

For 100% efficient of cooler at inter-stage, the work of compression in multistage compressor of n-stages is;

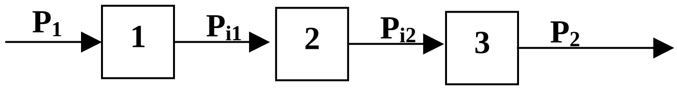
$$W = P_1V_1\frac{n\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right] \Rightarrow W = (101.3 \times 10^3 \times 0.82)\frac{2(1.25)}{1.25 - 1}\left[(20.4)^{(0.25)/2.5} - 1\right] = 292.35\frac{kJ}{kg}$$

Example -8.15-

A three stages compressor is required to compress air from 140 kPa and 283 K to 4000 kPa. Calculate the ideal intermediate pressures, the work required per kg of gas, and the isothermal efficiency of the process. Assume the compression to be adiabatic and perfect the inter-stage cooling to cool the air to the initial temperature. Taken that $\gamma = 1.4$.

Solution:

$$\frac{P_{i1}}{P_1} = \frac{P_{i2}}{P_{i1}} = \frac{P_2}{P_{i2}} = r = \left(\frac{P_2}{P_1}\right)^{\frac{1}{3}} = \left(\frac{4000}{140}\right)^{\frac{1}{3}} = 3.057$$



$\Rightarrow$ P_{i1}=3.057 (140) =428 kPa

P_{i2}=3.057 (428) =1308.4 kPa

$$W = P_1V_1 \frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right]$$

$$P_1V_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 283\text{K}}{(29\ \text{kg/kmol})} = 81.133 \quad (kJ/kg)$$

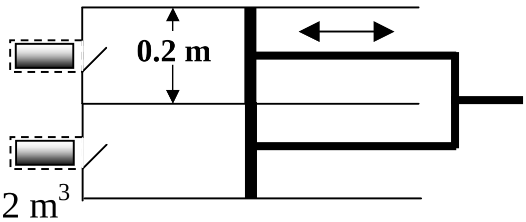
$$\Rightarrow W = 81.133\frac{3(1.4)}{0.4}\left[\left(\frac{4000}{140}\right)^{0.4/4.2} - 1\right] = 320.43\ kJ/kg$$

For isothermal compression = $W_{iso} = P_1V_1 \ln(P_2/P_1)$ = 272 kJ/kg

Isothermal efficiency = (W_{iso}/ W) 100 = 84.88 %

Example -8.16-

A twin-cylinder single-acting compressor, working at 5 Hz, delivers air at 515 kPa pressure at the rate of 0.2 m^3/s. If the diameter of the cylinder is 20 cm, the cylinder clearance ratio 5%, and the temperature of the inlet air 283 K, calculate the length of stroke of the piston and delivery temperature (γ=1.4).



Solution:

$T_2/T_1 = (P_2/P_1)^{(\gamma-1)/\gamma}$ $\Rightarrow$ $T_2=283(515/101.3)^{0.4/1.4}$=450K

The volume handled per cylinder = 0.2/2 =0.1 m^3/s

Volume per stroke per cylinder = (0.1 m^3/s) / (5 s^{-1}) = 0.02 m^3

Volume at inlet conditions = (0.02 m^3) (283/450) (515/101.3) = 0.0639 m^3

$V_1 - V_4 = V_s\,[1+ C - C\,(P_2/P_1)^{1/\gamma}]$ $\Rightarrow$ $0.0639 = V_s\,[1+ 0.05 - 0.05\,(515/101.3)^{1/1.4}]$

$\Rightarrow$ $V_s = 0.0718\ m^3 = \pi/4\ (0.2)^2\ L_{stroke}$ $\Rightarrow$ L_{stroke} = 2.286 m

Example -8.17-

In a single-acting compressor suction pressure and temperature are 101.3 kPa and 283 K, the final pressure is 380 kPa. If the compression is adiabatic and each new charge is heated 18 K by contact with the clearance gases, calculate the maximum temperature attained in the cylinder (γ=1.4).

Solution: On the first stroke the air enters at 283 K and is compressed adiabatically

$\Rightarrow$ $T_2 = 283\ (380/101.3)^{0.4/1.4}$ = 415 K

The clearance volume gases at 413 K which remain in the cylinder are able to raise the next cylinder full of air by 18 K i.e. the air temperature in the next cylinder is [18 + 283 = 301 K] $\Rightarrow$ The exit temperature = 301 $(380/101.3)^{0.4/1.4}$ = 439.2 K

On each subsequent stroke T_{in}=283 K, $T_{cylinder}$ = 301 K, and T_{exit} = 439.2 K.

Example -8.18-

A single-stage double-acting compressor running at 3 Hz is used to compess air from 110 kPa and 282 K to 1150 kPa. If the internal diameter of the cylinder 20 cm, the length of the stroke 25 cm, and the piston clearance 5%. Calculate;

a- The maximum capacity of machine, referred to air at initial conditions;

b- The theoretical power requirements under isentropic conditions.

Solution:

The swept volume per stroke = $2[\pi/4\,(0.2)^2\,(0.25)] = 0.0157\ m^3$

$(V_1 - V_4) = V_s[1 + C - C\,(P_2/P_1)^{1/\gamma}] \Rightarrow (V_1 - V_4) = 0.0157[1 + 0.05 - 0.05\,(1150/110)^{1/1.4}]$

$\Rightarrow \quad (V_1 - V_4) = 0.0123\ m^3$

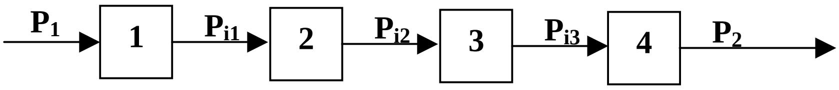
$$W = P_1(V_1 - V_4)\frac{\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] \Rightarrow W = 110(0.0123)\frac{1.4}{0.4}\left[\left(\frac{1150}{110}\right)^{0.4/1.4} - 1\right] = 5.775\ kJ/stroke$$

The power required = (3 stroke/s)(5.775 kJ/stroke) = 17.324 kW

Capacity = (3 stroke/s) (0.0123 m^3/stroke) = 0.0369 m^3/s

Example -8.19-

Methane is to be compressed fom atmospheric pressure ton 30 MPa in four stages. Calculate the ideal intermediate pressures and the work required per kg of gas. Assume compression to be isentropic and the gas to behave as an ideal gas and the initial condition at STP (γ=1.4).



Solution:

$$\frac{P_{i1}}{P_1} = \frac{P_{i2}}{P_{i1}} = \frac{P_{i3}}{P_{i2}} = \frac{P_2}{P_{i3}} = r = \left(\frac{P_2}{P_1}\right)^{\frac{1}{4}} = \left(\frac{30}{0.1013}\right)^{\frac{1}{4}} = 4.148$$

$\Rightarrow \quad P_{i1}$=4.148 (101.3 kPa) = 420.23 kPa

P_{i2}=4.148 (420.23 kPa) = 1743.27 kPa

P_{i3}=4.148 (1743.27 kPa) = 7231.75 kPa

P_2=4.148 (7231.75 kPa) = 30,000 kPa

$$W = P_1 V_1 \frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right]$$

$$(P_1V_1)_{STP} = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\,273\text{K}}{(16\ \text{kg/kmol})} = 141.857 \quad (kJ/kg)$$

$$\Rightarrow W = 141.857\frac{4(1.4)}{0.4}\left[\left(\frac{30{,}000}{101.3}\right)^{0.4/5.6} - 1\right] = 996.06\ kJ/kg$$