9.3 Dimensionless Groups for Mixing

Some of the various types of forces that may be arise during mixing or agitation will be formulated: -

1- Inertial Force [F_i]

Is associated with the reluctance of a body to change its state of rest or motion.

The inertial force (F_i) = (mass) (acceleration) = m.a

dF_i = dm (du/dt)

but m = ρ V = ρ A L

⇒ dm = ρ dV = ρ A dL

and u = dL/dt

$$\boxed{\textbf{Or}\ \dot{m} = \frac{dm}{dt} \Rightarrow dm = \dot{m}\,dt = \rho A\,u\,dt}$$

⇒ dF_i = ρ A dL du/dt = ρ A (dL/dt) du = ρ A u du

$$\Rightarrow \quad F_i = \int_0^{F_i} dF_i = \int_0^{u} \rho A u\,du = \rho A u^2/2$$

In mixing applications;

$A \propto D_A^2$ D_A: diameter of agitator

$u = \pi D_A N$ N: rotational speed

Therefore, the expression for inertial force may be written as;

$$\boxed{F_i \propto \rho D_A^4 N^2}$$

2- Viscous Force [F_v]

The viscous force for Newtonian fluid is given by:

F_v = μ A (du/dy)

In mixing applications;

$A \propto D_A^2$; $du/dy \propto \pi D_A N / D_A$

Therefore, the expression for viscous force may be written as;

$$\boxed{F_v \propto \mu D_A^2 N}$$

3- Gravity Force [F_g]

The inertial force (F_g) = (mass) (gravitational acceleration) = m.g

In mixing applications;

$m = \rho V = \rho A L \propto \rho D_A^3$

$$\boxed{F_g \propto \rho D_A^3 g}$$

4- Surface Tension Force [F_σ]

In mixing applications;

$$\boxed{F_\sigma \propto \sigma D_A}$$

In the design of liquid mixing systems the following dimensionless groups are of importance: -

1- The Power Number (Np)

$$\boxed{Np = \frac{P_A}{\rho N^3 D_A^5}}$$

where, P_A: is the power consumption.

2- The Reynolds Number (Re)$_m$

$$\boxed{(\mathrm{Re})_m = \frac{\text{Inertial Force}}{\text{Viscous Force}}} = \frac{F_i}{F_v} = \frac{\rho D_A^4 N^2}{\mu D_A^2 N}$$

$$\boxed{\Rightarrow \quad (\mathrm{Re})_m = \frac{\rho N D_A^2}{\mu}}$$

3- The Froude Number (Fr)$_m$

This number related to fluid surface [related to vortex system in mixing]

$$\boxed{(Fr)_m = \frac{\text{Inertial Force}}{\text{Gravity Force}}} = \frac{F_i}{F_g} = \frac{\rho D_A^4 N^2}{\rho D_A^3 g}$$

$$\boxed{\Rightarrow \quad (Fr)_m = \frac{N^2 D_A}{g}}$$

4- The Weber Number (We)$_m$

This number related to multiphase fluids [**or** fluid flow with interfacial forces]

$$\boxed{(We)_m = \frac{\text{Inertial Force}}{\text{Surface Tension Force}}} = \frac{F_i}{F_\sigma} = \frac{\rho D_A^4 N^2}{\sigma D_A}$$

$$\boxed{\Rightarrow \quad (We)_m = \frac{\rho N^2 D_A^3}{\sigma}}$$

It can be shown by **dimensional analysis** that the power number (Np) can be related to the Reynolds number (Re)$_m$ and the Froude number (Fr)$_m$ by the equation;

$$\boxed{Np = C(\mathrm{Re})_m^x (Fr)_m^y}$$

where, C is an overall dimensionless shape factor which represents the geometry of the system.

The last equation can also be written in the form;

$$\boxed{\Phi = \frac{Np}{(Fr)_m^y} = C(\mathrm{Re})_m^x}$$

where, Φ is defined as the dimensionless power function.

The Froude number (Fr)$_m$ is usually important only in situations where gross vortexing. Since vortexing is a gravitational effect, the (Fr)$_m$ is not required to describe a baffled liquid mixing systems. In this case the exponent of (Fr)$_m$ (i.e. y) in the last two equations is zero.[$(Fr)^y = (Fr)^0 = 1 \quad \Rightarrow [\Phi = Np]$.

Thus the non-vortexing systems, the equation of power function (Φ) can be written wither as;

$\boxed{\Phi = Np = C(\mathrm{Re})_m^x}$ or as; $\boxed{\log \Phi = \log Np = \log C + x \log(\mathrm{Re})_m}$

The Weber number of mixing (We)$_m$ is only of importance *when separate physical phases are present in the liquid mixing system* as in liquid-liquid extraction.