9.4 <u>Power Curve</u>

A power curve is a plot of the power function (Φ) or the power number (Np) against the Reynolds number of mixing $(Re)_m$ on log-log coordinates. <u>Each **geometrical configuration** has its own power curve</u> and since the plot involves dimensionless groups it is independent of tank size. <u>Thus a power curve that used to correlate power data in a 1 m^3 tank system is also valid for a 1000 m^3 tank system</u> provided that both tank systems have the same **<u>geometrical configuration</u>**.

The Figure below shows the power curve for the standard tank configuration. Since this is a baffled tank (non-vortexing system), the following equation is applied;

$$\log \Phi = \log Np = \log C + x \log (Re)_m \qquad \text{--------------------------- } (☼)$$

From the Figure it is clear that the power curve for the standard tank configuration is **linear** <u>in the laminar flow region (line-AB)</u> with slope (-1) in this region [$(Re)_m < 10$]. Then the last equation can be written in the following form;

$$\log \Phi = \log Np = \log C - \log (Re)_m$$

which can be rearranged to give;

$$\boxed{P_A = C\, \mu\, N^2\, D_A^3}$$

C is a constant depend on the type of agitator and vessel arrangement and if the tank is with or without baffles. **For the standard tank configuration <u>C = 71</u>** and for marine type 3-blade C = 41. Thus for **the laminar flow**, power (P_A) is <u>directly proportional</u> to *dynamic viscosity* (μ) *for a fixed agitator speed* (N).

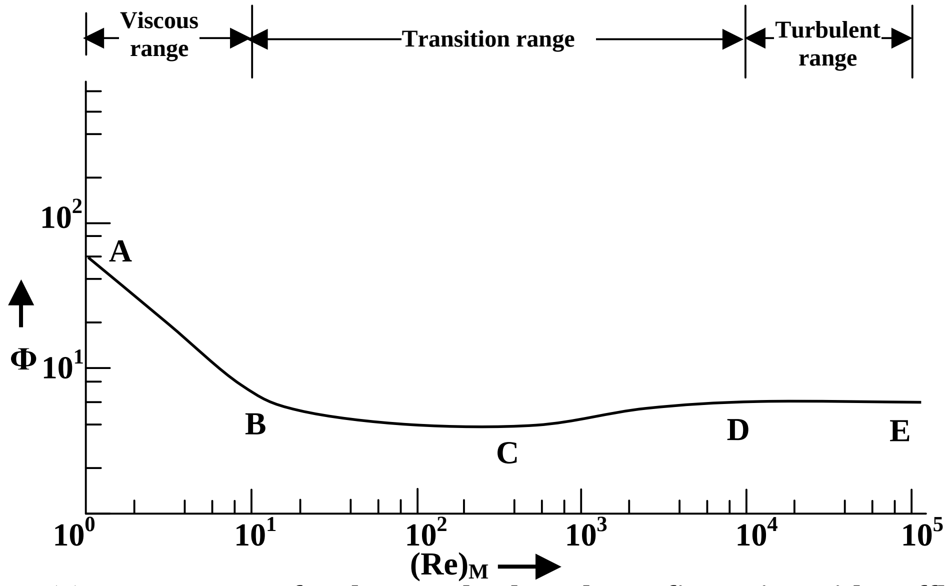

Figure (1): Power Curve for the Standard Tank Configuration with Baffles

For **the transition flow region BCD** which extends up to $(Re)_m = 10{,}000$, the constant (C) and the slope (x) in equation (☼) vary continuously.

In **fully turbulent flow** $(Re)_m > 10{,}000$, the curve becomes horizontal and the power function (Φ) is independent of Reynolds number of mixing $(Re)_m$.

i.e. $\boxed{\Phi = Np = 6.3 \qquad \text{for } (Re)_m > 10{,}000}$

At point (C) on the power curve, for the standard tank configuration, <u>enough energy</u> is being transferred to the liquid for *vortexing* to start. However the baffles in the tank prevent this.

If the baffles were not present, vortexing would develop and the power curve would be as shown in Figure below;

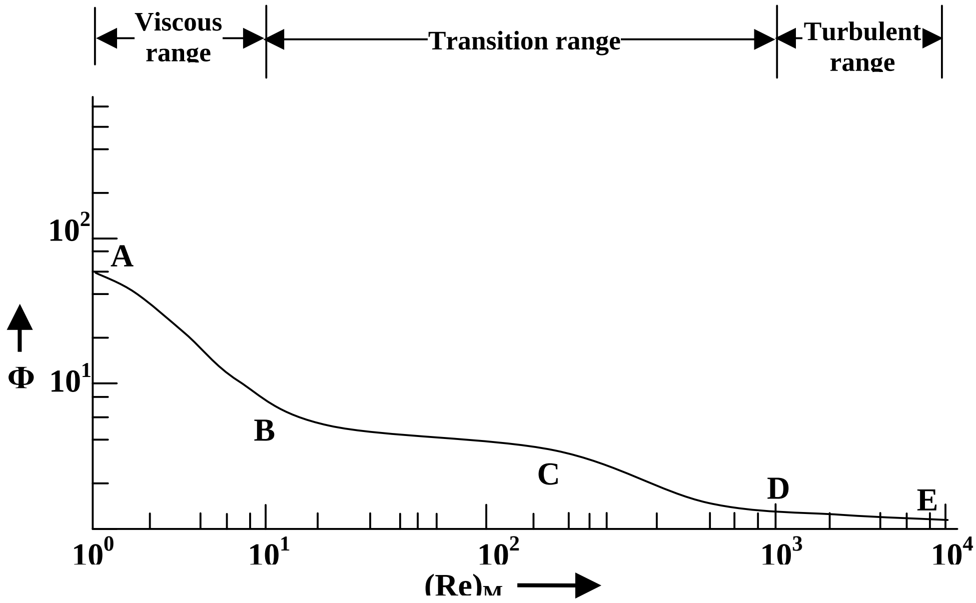

Figure (1): Power Curve for the Standard Tank Configuration without Baffles

The power curve for the baffled system is identical with that for the unbaffled system up to point (C) where [$(\mathrm{Re})_m \approx 300$]. As the $(\mathrm{Re})_m$ increases beyond point (C) in the unbaffled system, vortexing increases and the power falls sharply as shown in the above Figure.

As mentioned previously it can be shown by **<u>dimensional analysis</u>** that the power number (Np) can be related to the Reynolds number $(\mathrm{Re})_m$ and the Froude number $(\mathrm{Fr})_m$ by the equation;

$$Np = C(\mathrm{Re})_m^x (Fr)_m^y$$

$$= \quad \log Np = \log C + x\log(\mathrm{Re})_m + y\log(Fr)_m$$

<u>For the unbaffled system</u>

	$\Phi = Np$	for $(\mathrm{Re})_m < 300$
And	$\Phi = Np/[(\mathrm{Fr})_m]^y$	for $(\mathrm{Re})_m > 300$

A plot of (Np) against $(\mathrm{Fr})_m$ on log-log coordinates is a straight line of slop y at a constant $(\mathrm{Re})_m$. A number of lines can be plotted for different values of $(\mathrm{Re})_m$. A plot of (y) against $\log(\mathrm{Re})_m$ is also a straight line. If the slope of the line is $(-1/\beta)$ and the intercept at $(\mathrm{Re})_m = 1$ is (α/β) then

$$\boxed{y = \frac{\alpha - \log(\mathrm{Re})_m}{\beta}}$$

$$\boxed{\therefore \quad \Phi = \frac{Np}{(Fr)_m^y} = \frac{Np}{\left[(Fr)_m\right]^{\frac{\alpha - \log(\mathrm{Re})_m}{\beta}}}}$$

The values of (α and β) are varying for various vortexing system. For a 6-blade flat blade turbine agitator of 0.1 m diameter [(α = 1) and (β = 40)]

If a power curve is available for particular system geometry, it can be used to calculate the power consumed by an agitator at various rotational speeds, liquid viscosities and densities. The procedure is as follows: -

1- Calculate $(Re)_m$
2- Read power number (Np) or power function (Φ) from the appropriate power curve
3- Calculate the power (P_A) from
either $\boxed{P_A = Np\, \rho\, N^3\, D_A^5}$ or $\boxed{P_A = \Phi\, [(Fr)_m]^y\, \rho\, N^3\, D_A^5}$

These equations can b used to calculate only the power consumed by the agitator. Electrical and mechanical losses required additional power, which occur in all mixing system.

Example -9.1-

Calculate the theoretical power in Watt for a 3 m diameter, 6-blade flat blade turbine agitator running at 0.2 rev/s in a tank system conforming to the standard tank configuration. The liquid in the tank has a dynamic viscosity of 1 Pa.s, and a liquid density of 1000 kg/m^3.

Solution:

$(Re)_m = \rho\, N\, D_A^2 / \mu = (1000)\,(0.2)\,(3)^2 / 1 = 1{,}800$

From Figure (1) $\Phi = Np = 4.5$

The theoretical power for mixing

$$P_A = Np\, \rho\, N^3\, D_A^5$$
$$= 4.5\,(1000)\,(0.2)^3\,(3)^5$$
$$= 8{,}748 \text{ W}$$

Example -9.2-

Calculate the theoretical power in Watt for a 0.1 m diameter, 6-blade flat blade turbine agitator running at 16 rev/s in a tank system without baffles and conforming to the standard tank configuration. The liquid in the tank has a dynamic viscosity of 0.08 Pa.s, and a liquid density of 900 kg/m^3.

Solution:

$(Re)_m = \rho\, N\, D_A^2 / \mu = (900)\,(16)\,(0.1)^2 / (0.08) = 1{,}800$

From Figure (2) $\Phi = 2.2$

The theoretical power for mixing

$$P_A = \Phi\, [(Fr)_m]^y\, \rho\, N^3\, D_A^5$$

$$y = \frac{\alpha - \log(Re)_m}{\beta} \quad \Rightarrow y = \frac{1 - \log(1800)}{40} = -0.05638$$

$$(Fr)_m = N^2\, D_A / g = (16)^2\,(0.1) / 9.81 = 2.61$$
$$[(Fr)_m]^y = [2.61]^{-0.05638} = 0.9479$$

$$\Rightarrow P_A = 2.2\,(0.9479)\,(900)\,(16)^3\,(0.1)^5$$
$$= 76.88 \text{ W}$$