# 9 Fluid motion in the presence of solid particles

## 9.1 Relative motion between a fluid and a single particle

Consider the relative motion between a particle and an infinitely large volume of fluid. Since only the relative motion is considered the following cases are covered:

1 a stationary particle in a moving fluid;
2 a moving particle in a stationary fluid;
3 a particle and a fluid moving in opposite directions;
4 a particle and a fluid both moving in the same direction but at different velocities.

In contrast to single-phase flow in a pipe of constant cross section, flow around a sphere or other bluff object exhibits several different flow regimes at different values of the Reynolds number.

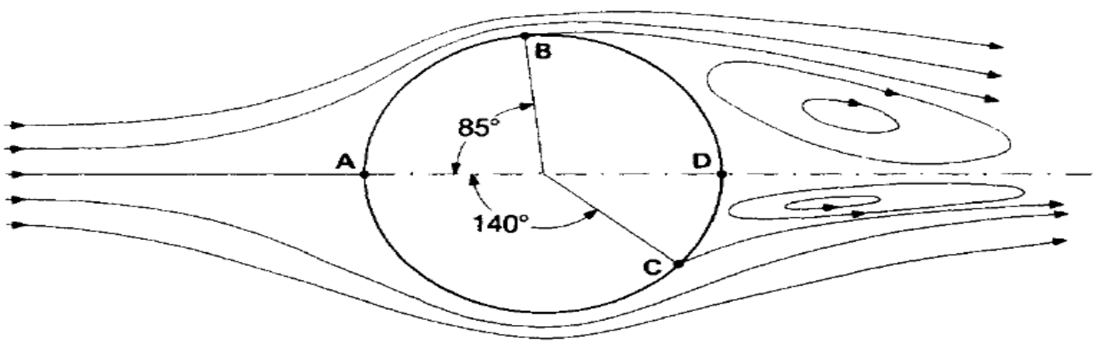
For flow around a spherical particle of diameter $d_p$, the appropriate definition of the Reynolds number is

$$Re_p = \frac{\rho u_p d_p}{\mu} \tag{9.1}$$

where $u_p$ is the speed of the particle relative to the fluid.

Except at very low values of the particle Reynolds number, a wake forms behind the sphere as shown in Figure 9.1. The upper half of this composite diagram shows the streamlines for flow at an intermediate value of $Re_p$, while the lower half shows the streamlines for a higher value of $Re_p$. In general, separation of the flow from the surface of the sphere occurs over the rear part creating a large low pressure wake shown by the recirculating flow. The presence of this low pressure wake is responsible for most of the drag when flow separation occurs.

At all values of the particle Reynolds number $Re_p$, the fluid is brought to rest relative to the particle at A, which is therefore a stagnation point where the pressure is higher than in the flowing fluid (see equation 1.19 in



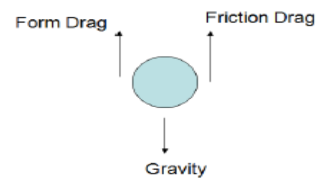




Figure 9.1
*Streamlines for flow around a sphere*
Upper half: intermediate $Re_p$ giving a laminar boundary layer
Lower half: high $Re_p$ giving a turbulent boundary layer

Section 1.5). In flowing round the sphere, the fluid has to accelerate and therefore, by Bernoulli's equation, the pressure falls towards the mid-point of the sphere's surface.

At very low values of $Re_p$, when the flow is dominated by viscous stresses, the fluid creeps round the rear of the sphere and no separation occurs: the flow is symmetrical fore and aft. In this case the drag force on the particle is due solely to the shear stress generated by the fluid's velocity gradient normal to the surface. This force is often known as skin friction. When no separation occurs, the highest velocity of the fluid occurs over the mid-point of the sphere's surface and consequently the pressure here is a minimum. As the fluid decelerates over the rear half of the sphere, pressure recovery occurs. Thus, the flow in this region is in the direction of increasing pressure: this is known as an adverse pressure gradient and it tends to make the flow unstable at higher values of the Reynolds number.

As the Reynolds number is increased, fluid inertia becomes more significant. In addition, the pressure at point D increases and, instead of flowing round the rear surface of the sphere, the fluid is pushed away so that flow separation occurs as shown in the upper half of Figure 9.1. Separation occurs at point B. The whole of the wake is a region of relatively low pressure, very close to that at the point of separation, and much lower than the pressure near point A. The force arising from this pressure difference is known as form drag because it is due to the (bluff) shape of the particle. The total drag force is a combination of skin friction and form drag. In this flow regime a laminar boundary layer is formed over the surface of the sphere from A to B.

On increasing the Reynolds number further, a point is reached when the boundary layer becomes turbulent and the point of separation moves further back on the surface of the sphere. This is the case illustrated in the lower half of Figure 9.1 with separation occurring at point C. Although there is still a low pressure wake, it covers a smaller fraction of the sphere's surface and the drag force is lower than it would be if the boundary layer were laminar at the same value of $Re_p$.

Roughening the surface of a sphere causes the transition to a turbulent boundary layer to occur at a lower value of the Reynolds number. This explains the apparent anomaly that, at certain values of the Reynolds number, the drag will be lower for a sphere with a rough surface than for a similar sphere with a smooth surface. It is for the same reason that golf balls are made with a dimpled surface.

Consider a spherical particle of diameter $d_p$ and density $\rho_p$ falling with a velocity $u_p$ under the influence of gravity in a fluid of density $\rho$. The net gravitational force $F_1$ on the particle is given by the equation

$$F_1 = \frac{\pi d_p^3}{6}(\rho_p - \rho)g \tag{9.2}$$

where $\pi d_p^3/6$ is the volume of the spherical particle.

The retarding force $F_2$ on the particle from the fluid is given by the equation

$$F_2 = C_d S_p \frac{\rho u_p^2}{2} \tag{9.3}$$

where $C_d$ is a dimensionless drag coefficient and $S_p$ is the projected area of the particle in a plane perpendicular to the direction of the fluid stream. Equation 9.3 may be obtained by dimensional analysis. The drag coefficient has a role similar to the friction factor for flow in pipes.

For steady flow the forces $F_1$ and $F_2$ are equal and opposite and the particle reaches a constant speed $u_t$. Equations 9.2 and 9.3 can be combined and written as

$$\frac{\pi d_p^3}{6}(\rho_p - \rho)g = C_d S_p \frac{\rho u_t^2}{2} \tag{9.4}$$

For a spherical particle $S_p = \pi d_p^2/4$
rewritten in the form

$$u_t = \sqrt{\frac{4 d_p(\rho_p - \rho)g}{3 C_d \rho}} \tag{9.5}$$

where $u_t$ is known as the terminal settling or falling velocity.