As a result of the changing flow patterns described above, the drag coefficient C_d is a function of the Reynolds number. For the streamline flow range of Reynolds numbers, $Re_p < 0.2$, the drag force F_2 is given by.

$$F_2 = 3\pi d_p \mu u_t \tag{9.6}$$

and consequently, from equation 9.3, the drag coefficient is

$$C_d = \frac{24}{Re_p} \tag{9.7}$$

From equations 9.5 and 9.7, the terminal falling velocity u_t for the streamline flow range of Reynolds numbers is given by

$$u_t = \frac{d_p^2(\rho_p - \rho)g}{18\mu} \tag{9.8}$$

Equations 9.6 and 9.8 were derived by Stokes and are known as Stokes's equations for steady creeping flow round a sphere.

For the Reynolds number range $0.2 < Re_p < 500$, it has been shown that

$$C_d = \frac{24}{Re_p}(1 + 0.15 Re_p^{0.687}) \tag{9.9}$$

Equation 9.8 is an empirical equation which is only approximately true. Over the more limited range $2 < Re_p < 500$ a simpler equation is adequate:

$$C_d = \frac{18.5}{Re_p^{0.6}} \tag{9.10}$$

This is particularly useful because equation 9.7 for the Stokes regime can be extended to $Re = 2$ with negligible error.

For the Reynolds number range $500 < Re_p < 200000$

$$C_d = 0.44 \tag{9.11}$$

When the Reynolds number Re_p reaches a value of about 300000, transition from a laminar to a turbulent boundary layer occurs and the point of separation moves towards the rear of the sphere as discussed above. As a result, the drag coefficient suddenly falls to a value of 0.10 and remains constant at this value at higher values of Re_p.

For the most part, solid particles in fluid streams have Reynolds numbers which are much lower than 500.

Pettyjohn and Christiansen (1948) gave equations for the terminal settling velocities of particles which deviate from a spherical shape.

Lapple and Shepherd (1940) presented plots of the dimensional drag

coefficient C_d against the particle Reynolds number for spheres, discs and cylinders.

Equation 9.5 gives the terminal settling velocity for a spherical particle. For a non-spherical particle, equation 9.5 can be written in the modified form

$$u_t = \sqrt{\frac{4d_p\psi(\rho_p-\rho)g}{3C_d\rho}} \tag{9.12}$$

For a spherical particle the dimensionless correction factor $\psi = 1$ and equations 9.5 and 9.12 become identical.

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9.2 Relative motion between a fluid and a concentration of particles

So far the relative motion between a fluid and a single particle has been considered. This process is called free settling. When a fluid contains a concentration of particles in a vessel, the settling of an individual particle may be hindered by the other particles and by the walls. When this is the case, the process is called hindered settling. Interference is negligible if the particles are at least 10 to 20 diameters away from each other and the vessel wall [Larian (1958)]. In this case the particles can be considered to be free settling.

Hindered settling results from collisions between particles and also between particles and the wall. In addition high particle concentrations reduce the flow area and increase the velocity of the fluid with a consequent decrease in settling rate. Furthermore particle concentrations increase the apparent density and dynamic viscosity of the fluid.

Richardson and Zaki (1954) showed that in the Reynolds number range $Re_p < 0.2$, the velocity u_c of a suspension of coarse spherical particles in water relative to a fixed horizontal plane is given by the equation

$$\frac{u_c}{u_t} = \varepsilon^{4.6} \tag{9.13}$$

where u_t is the terminal settling velocity for a single particle and ε is the voidage fraction of the suspension which is unity for a single particle in an infinite amount of fluid. The velocity of the particles relative to the liquid can be derived from equation 9.13, as explained in Section 7.3.

Settling can be used to classify or separate particles since different sized particles settle at different velocities. Similarly elutriation can also be used to classify particles where small particles are carried upwards with the