fluid and large particles sink. Particles of the same size but of different densities can also be separated by settling or elutriation.

Consider two spherical particles 1 and 2 of the same diameter but of different densities settling freely in a fluid of density ρ in the streamline Reynolds number range $Re_p < 0.2$. The ratio of the terminal settling velocities u_{t1}/u_{t2} is given by equation 9.8 rewritten in the form

$$\frac{u_{t1}}{u_{t2}} = \frac{\rho_{p1} - \rho}{\rho_{p2} - \rho} \tag{9.14}$$

The greater the ratio u_{t1}/u_{t2} the greater the ease of separation. Thus the fluid density ρ can be chosen to give a high ratio of terminal settling velocities.

Similarly, for two spherical particles 1 and 2 of the same density ρ_p but of different diameters settling freely in a fluid of density ρ in the streamline Reynolds number range $Re_p < 0.2$, the ratio of the terminal settling velocities u_{t1}/u_{t2} is given by equation 9.8 rewritten in the form

$$\frac{u_{t1}}{u_{t2}} = \left(\frac{d_{p1}}{d_{p2}}\right)^2 \tag{9.15}$$

where d_{p1}/d_{p2} is the ratio of the particle diameters.

Similarly, from equation 9 8, two particles will settle at the same speed in the same fluid in the streamline flow regime if their densities and diameters are related by equation 9.16:

$$\frac{d_{p1}}{d_{p2}} = \sqrt{\left(\frac{\rho_{p2} - \rho}{\rho_{p1} - \rho}\right)} \tag{9.16}$$

The classification or separation of particles can be carried out more rapidly in centrifugal separators than in gravity settlers. In gravity settlers, the particles travel vertically downwards, whereas in centrifugal separators the particles travel radially outwards. A particle of mass m rotating at a radius r with an angular velocity ω is subject to a centripetal force $mr\omega^2$ which can be made very much greater than the vertically directed gravity force mg.

The terminal settling velocity u_t for a single spherical particle in a centrifugal separator can be calculated from equation 9.5 with the centripetal acceleration $r\omega^2$ replacing the gravitational acceleration g to give

$$u_t = \sqrt{\frac{4d_p(\rho_p - \rho)r\omega^2}{3C_d\rho}} \tag{9.17}$$

A very small particle may still be in laminar flow in a centrifugal separator. In this case $r\omega^2$ may be written in place of g in equation 9.8 to give

$$u_t = \frac{d_p^2(\rho_p - \rho)r\omega^2}{18\mu} \tag{9.18}$$

In addition to hydrodynamic interactions between solid particles and a fluid, physico-chemical forces may act between pairs of particles. These forces tend to form a structure which prevents the particles from settling out [Cheng (1970)]. If the forces are sufficiently strong a homogeneous slurry results which usually has non-Newtonian rheological characteristics. If the structure is weak, the slurry is shear thinning. Slurries with a high proportion of solids tend to be shear thickening.

Einstein studied homogeneous slurries of spherical particles in a liquid of the same density. He showed that the distortion of the streamlines around the particles caused the dynamic viscosity of the slurry to increase according to the equation

$$\mu = \mu_L(1 + 2.5\alpha) \tag{9.19}$$

where μ and μ_L are the dynamic viscosities of the slurry and liquid respectively and α is the volume fraction of the solids. Equation 9.19 holds for low concentrations up to $\alpha = 0.02$.

Note that $\alpha = 1 - \varepsilon$.