9.3 Fluid flow through packed beds

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In a packed bed of unit volume, the volumes occupied by the voids and the solid particles are ε and $(1-\varepsilon)$ respectively where ε is the voidage fraction or porosity of the bed. Let S_o be the surface area per unit volume of the solid material in the bed. Thus the total surface area in a packed bed of unit volume is $(1-\varepsilon)S_o$.

For a spherical particle of diameter d_p the value of $S_o = 6/d_p$. For a non-spherical particle with an average particle diameter d_p, the value $S_o = 6/(d_p \psi)$ where $\psi = 1$ for a spherical particle. Values of ψ for other shapes are readily available [Perry (1984)].

An equivalent diameter d_e for flow through the bed can be defined as four times the cross-sectional flow area divided by the appropriate flow perimeter. For a random packing, this is equal to four times the volume occupied by the fluid divided by the surface area of particles in contact with the fluid.

Thus, the equivalent diameter is

$$d_e = \frac{4\varepsilon}{(1-\varepsilon)S_o} \tag{9.20}$$

The velocity calculated by dividing the volumetric flow rate by the whole cross-sectional area of the bed is known as the superficial velocity u. The mean velocity within the interstices of the bed is then $u_b = u/\varepsilon$.

A Reynolds number for flow through a packed bed can be defined as

$$Re_b = \frac{\rho u_b d_e}{\mu} \tag{9.21}$$

which when combined with equation 9.20 can be written as

$$Re_b = \frac{4\rho u}{\mu(1-\varepsilon)S_o} \tag{9.22}$$

An alternative Reynolds number has been used to correlate data and is defined as

$$Re'_b = \frac{\rho u}{\mu(1-\varepsilon)S_o} \tag{9.23}$$

For a packed bed consisting of spherical particles, equation 9.23 can be written in the form

$$Re'_b = \frac{\rho u d_p}{6\mu(1-\varepsilon)} \tag{9.24}$$

The corresponding equation for non-spherical particles is

$$Re'_b = \frac{\rho u d_p \psi}{6\mu(1-\varepsilon)} \tag{9.25}$$

Consider fluid flowing steadily through a packed bed of height L and unit cross-sectional area. A pressure drop ΔP_f occurs in the bed because of frictional viscous and drag forces. Let the resistance per unit area of surface be τ_b. A force balance across unit cross-sectional area gives

$$\Delta P_f \varepsilon = \tau_b L(1-\varepsilon)S_o \tag{9.26}$$

$$\frac{f_b}{2} = \frac{\tau_b}{\rho u_b^2} = \left(\frac{\Delta P_f}{L}\right)\left[\frac{\varepsilon^3}{(1-\varepsilon)S_o \rho u^2}\right] \tag{9.28}$$

where f_b is a dimensionless friction factor for flow through a packed bed. Various other definitions of friction factors for flow in packed beds have been used [Longwell (1966)].

For laminar flow where $Re'_b \leq 2$

$$\frac{f_b}{2} = \frac{5}{Re'_b} \tag{9.29}$$

The transition to turbulent flow is gradual. Turbulence commences initially in the largest channels and eventually extends to the smaller channels.

For the complete range of Reynolds number Carman (1937) gave the equation

$$\frac{f_b}{2} = \frac{5}{Re'_b} + \frac{0.4}{(Re'_b)^{0.1}} \tag{9.30}$$

Log log plots of f_b against Re'_b are readily available [Perry (1984)] for randomly packed beds.

The Hagen–Poiseuille equation for steady laminar flow of Newtonian fluids in pipes and tubes can be written as

$$u = \left(\frac{\Delta P_f}{L}\right)\frac{d_i^2}{32\mu} \tag{1.65}$$

For a packed bed, substituting the equivalent diameter d_e from equation 9.20 into equation 1.65 gives

$$u_b = \left(\frac{\Delta P_f}{L}\right)\left(\frac{1}{32\mu}\right)\left[\frac{16\varepsilon^2}{(1-\varepsilon)^2 S_o^2}\right] \tag{9.31}$$

or, since $u_b = u/\varepsilon$

$$u = \left(\frac{\Delta P_f}{L}\right)\left(\frac{1}{2\mu}\right)\left[\frac{\varepsilon^3}{(1-\varepsilon)^2 S_o^2}\right] \tag{9.32}$$

Equation 9.32 does not hold for flow through packed beds and should be replaced by the equation

$$u = \left(\frac{\Delta P_f}{L}\right)\left(\frac{1}{K_c \mu}\right)\left[\frac{\varepsilon^3}{(1-\varepsilon)^2 S_o^2}\right] \tag{9.33}$$

which can also be written in the form