$$\Delta P_f = (K_c \mu L)\left[\frac{(1-\varepsilon)^2 S_o^2}{\varepsilon^3}\right] u \tag{9.34}$$

Equation 9.33 is the Carman–Kozeny equation [Carman (1937)]. The parameter K_c has a value which depends on the particle shape, the porosity and particle size range. The value lies in the range 3.5 to 5.5 but the value most commonly used is 5.

For spherical particles $S_o = 6/d_p$ and equation 9.34 can be written as

$$\Delta P_f = (180 \mu L)\left[\frac{(1-\varepsilon)^2}{\varepsilon^3 d_p^2}\right] u \tag{9.35}$$

where $K_c = 5.0$.

Example 9.1

A gas of density $\rho = 1.25\ \text{kg/m}^3$ and dynamic viscosity $\mu = 1.5 \times 10^{-5}$ Pa s flows steadily through a bed of spherical particles of diameter $d_p = 0.005$ m. The bed has a height of 3.00 m and a voidage of $\frac{1}{3}$. The superficial velocity $u = 0.03$ m/s. Calculate the Reynolds number and the frictional pressure drop over the bed.

Calculations

$$\text{Reynolds number } Re'_b = \frac{\rho u d_p}{6\mu(1-\varepsilon)} \tag{9.24}$$

Substituting the given values

$$Re'_b = \frac{(1.25\ \text{kg/m}^3)(0.03\ \text{m/s})(0.005\ \text{m})(3)}{(6)(1.50 \times 10^{-5}\ \text{Pa s})(2)}$$

$$= 3.125$$

The frictional pressure drop is given by

$$\Delta P_f = (180 \mu L)\left[\frac{(1-\varepsilon)^2}{\varepsilon^3 d_p^2}\right] u \tag{9.35}$$

Given that

$$(1-\varepsilon)^2 = \tfrac{4}{9}$$

$$\frac{(1-\varepsilon)^2}{\varepsilon^3} = 12$$

$$d_p^2 = 2.5 \times 10^{-5}\ \text{m}^2$$

$$u = 0.03\ \text{m/s}$$

$$\mu = 1.5 \times 10^{-5}\ \text{Pa s}$$

$$L = 3.0\ \text{m}$$

it follows that

$$\Delta P_f = (180)(1.5 \times 10^{-5}\ \text{Pa s})\,\frac{(3.0\ \text{m})(12)(0.03\ \text{m/s})}{(2.5 \times 10^{-5}\ \text{m}^2)}$$

$$= \underline{116.6\ \text{Pa}}$$