Minimum Fluidization

The frictional pressure drop at the point of minimum fluidization equalizes the bed mass per unit of cross-sectional area.

$$
\begin{aligned}
-\Delta P_{friction} &= 150\,\frac{U_{mf}\,\mu\left(1-\varepsilon_{mf}\right)^2 \Delta x_{mf}}{D_{sv}^2\,\varepsilon_{mf}^3} + 1.75\,\frac{U_{mf}^2\,\rho\left(1-\varepsilon_{mf}\right)\Delta x_{mf}}{D_{sv}\,\varepsilon_{mf}^3} \\
&= g\Delta x_{mf}\left(\rho_p-\rho\right)\left(1-\varepsilon_{mf}\right)
\end{aligned}
$$

The frictional pressure drop at the point of minimum fluidization (U_{mf} , ε_{mf} , Δx_{mf}), can be considered equal to the frictional pressure drop in a fixed bed (Ergun)

Minimum Fluidization Velocity (U_{mf})

Dimensionless relationship following from equation on previous slide

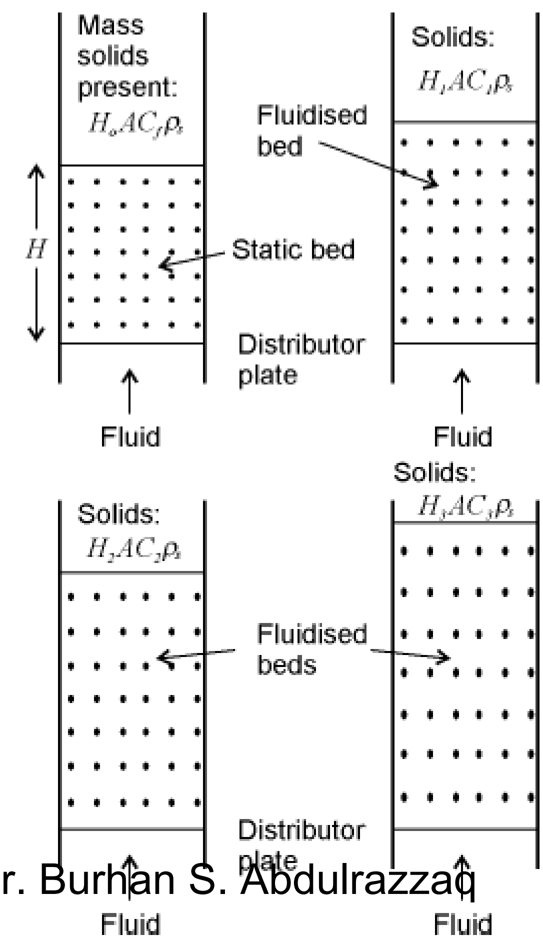
$$\mathrm{Re}_{mf} = \sqrt{C_1^2 + C_2 Ar} - C_1$$

$$Re_{mf} = U_{mf}\rho D_{sv}/\mu \qquad Ar = g\rho(\rho_p - \rho)D_{sv}^3/\mu^2$$

$$C_1 = 300(1-\varepsilon_{mf})/7 \qquad C_2 = \varepsilon_{mf}^3/1.75$$

Fluidization

The fluidization principle is straight forward: passing a fluid upwards through a packed bed of solids produces a pressure drop due to fluid drag. When the fluid drag force is equal to the bed weight the particles no longer rest on each other; this is the point of fluidization. The superficial velocity at this point is known as the 'minimum fluidizing velocity' (Umf). If the fluid velocity is increased further the pressure drop does not significantly increase – it remains equal to the bed weight per unit area, but the bed may expand; i.e. grow taller as illustrated in figure. Commercial gaseous fluidized beds are usually operated at flow rates many times that required for minimum fluidization, typically 5 to 20 times. Liquid



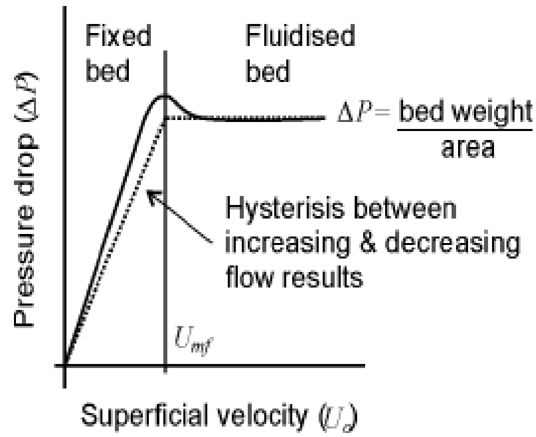
fluidized beds operate at values closer to Umf.
Material balance indicates that, in general

$$Ci = \frac{C_f H_0}{H_i}$$

The Fig. shows that up to the fluidization point pressure with superficial velocity is linear in accordance with Darcy's law:

$$\Delta P = \frac{\mu}{k} L U_o$$

On increasing the flow particles rearrange before fluidisation giving rise to the maximum – on decreasing flow a lower pressure drop may be found, giving a hysterisis.



Minimum fluidizing velocity

The fundamental equation for fluid flow through porous media, under laminar flow condition, is Darcy's law,

$$\frac{\Delta P}{L} = \frac{\mu}{k} \frac{dv}{dt} \frac{1}{A} \qquad - \ - - \ (*)$$

In most instances the Kozeny-Carman

$$\frac{\Delta P}{L} = \mu \left[\frac{k\,(1-\varepsilon)^2 S_v^2}{\varepsilon^3} \right] U_0 = \mu \left[\frac{k\,(1-\varepsilon)^2 S_v^2}{\varepsilon^3} \right] \frac{dv}{dt} \frac{1}{A} \qquad - \ - \ - - \ (**)$$

equation is preferred because it has an explicit expression for permeability in term on bed porosity and specific surface. The pressure at the base of the fluid (due to fluid weight) comes from the static component of Bernoulli's equation

$$\Delta P = L \rho g \qquad - - - - \qquad (1)$$

Where L is fluid height, g is the acceleration due to gravity and ρ is the fluid density. For a suspension, similar equation is valid but, for the static pressure due to the solid particle, we must take account of buoyancy and the proportion of particles present.

Clearly, if there are no solid present there would be zero static pressure due to solids but rather than use the solid concentration by volume fraction we use the porosity

$$\Delta P = (1-\varepsilon)\, g\, (\rho_s-\rho)\, L \qquad \text{- - -(2)}$$

Where ρ_s is the true solid density (kg/m^3). Combining equation 1 and * remembering that we started that fluidization occurs when the bed weight (per unit area) equals the fluid drag gives

$$U_{mf} = \frac{k}{\mu}\left(\rho_s-\rho\right) g\left(1-\varepsilon\right) \qquad \text{- - - - -(3)}$$

N. B. the minimum fluidization velocity is a superficial velocity (not interstitial). It is common to see equation 3 written with the permeability term expanded as provided by Kkozeny-Carman equation, see equation (**), and assuming spherical particles, equation

$$\frac{\Delta P}{L} = \mu \left[\frac{36\,K\left(1-\varepsilon\right)^2 S_v^2}{\varepsilon^3\, {x_{Sv}}^2}\right] U_0$$

, gives an alternative equation for minimum fluidizing velocity

Where

ΔP is the pressure drop over the bed

L is the bed depth

μ is the viscosity

ε is the bed porosity

Sv is the specific surface area per unit volume of the particles

U_0 is the fluid superficial velocity

$$U_{mf} = \frac{\left(\rho_s-\rho\right) g\, \varepsilon^3 {x_{Sv}}^2}{180\left(1-\varepsilon\right)\mu} \qquad \text{- - - - - - - - - - (4)}$$