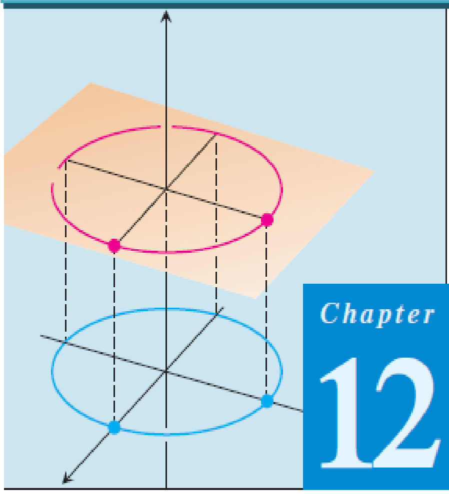
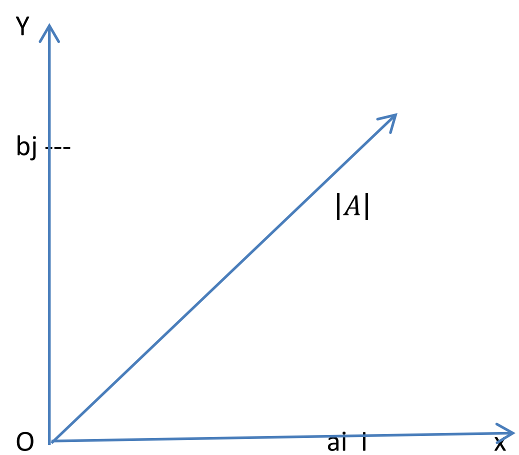
VECTORS AND THE GEOMETRY IN SPACE



Vectors and the Geometry of Space

OVERVIEW To apply calculus in many real-world situations and in higher mathematics, we need a mathematical description of three-dimensional space. In this chapter we introduce three-dimensional coordinate systems and vectors. Building on what we already know about coordinates in the *xy*-plane, we establish coordinates in space by adding a third axis that measures distance above and below the *xy*-plane. Vectors are used to study the analytic geometry of space, where they give simple ways to describe lines, planes, surfaces, and curves in space. We use these geometric ideas in the rest of the book to study motion in space and the calculus of functions of several variables, with their many important applications in science, engineering, economics, and higher mathematics.

Vectors in Plane



$$\overrightarrow{A} = \overrightarrow{OA} = ai + bj \qquad Vector$$

$$|\vec{A}| = \sqrt{a^2 + b^2} \qquad Length\ of\ Vector$$

$$\alpha + \beta = \frac{\pi}{2}$$

$$Cos\alpha = sin\beta$$

$$sin\alpha = Cos\beta$$

$$i, j \ \ are\ the\ fundamental\ unit\ vector$$

$$Unit\ Vector = \vec{U} = \frac{\vec{A}}{|\vec{A}|} = \frac{ai + bj}{\sqrt{a^2 + b^2}}$$

$$Note{:}\ |\vec{U}| = 1 \qquad\qquad Unit\ Vector$$

$$\therefore\ \vec{U} = cos\alpha\ i + sin\beta\ j \quad OR \quad \vec{U} = cos\alpha\ \beta + sin\alpha\ j$$

$$Where, \quad \cos\alpha = \frac{a}{|\vec{A}|} \quad \& \quad sin\alpha = \frac{b}{|\vec{A}|}$$

Vector Algebra Operation

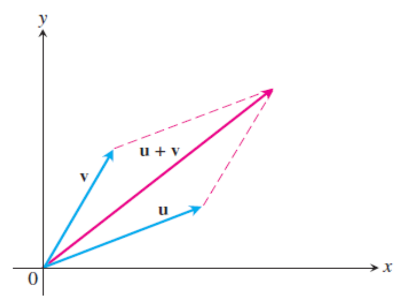
1- Vector Addition:

Let $\vec{v} = a_1 i + b_1 j$

and $\vec{u} = a_2 i + b_2 j$

$Then,$

$$\vec{v} + \vec{u} = (a_1 + a_2)i + (b_1 + b_2)j$$



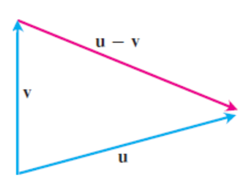
2- Vector Subtraction:

Let $\vec{v} = a_1 i + b_1 j$

and $\vec{u} = a_2 i + b_2 j$

$Then,$

$$\vec{v} + \vec{u} = (a_1 - a_2)i + (b_1 - b_2)j$$



3- *Vector Length:*

Let $\vec{v} = a\,i + b\,j$

$$\therefore\ |\vec{v}| = \sqrt{a^2 + b^2}$$

Examples:

1- $\boldsymbol{Let\ \vec{v} = 8i - j\quad and\ \vec{u} = 3i + 5j\ ,\ find}$:

$\vec{v} + \vec{u}\ ,\quad \vec{v} - \vec{u}\ ,\qquad |\vec{v}|\ ,|\vec{u}|\ ,\qquad \boldsymbol{unit\ vector\ of}\ (\vec{v})\ \&\ \boldsymbol{unit\ vector\ of}\ (\vec{u}).$

Sol.:

$$\vec{v} + \vec{u} = (8+3)i + (-1+5)j = 11i + 4j$$

$$\vec{v} - \vec{u} = (8-3)i + (-1-5)j = 5i - 6j$$

$$|\vec{v}| = \sqrt{a^2 + b^2} = \sqrt{8^2 + (-1)^2} = \sqrt{65}$$

$$|\vec{u}| = \sqrt{3^2 + 5^2} = \sqrt{34}$$

$$unit\ vector\ \ \vec{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{8i - j}{\sqrt{65}} = \frac{8}{\sqrt{65}}i - \frac{1}{\sqrt{65}}j$$

$$unit\ vector\ \ \vec{u} = \frac{\vec{u}}{|\vec{u}|} = \frac{3i + 5j}{\sqrt{34}} = \frac{3}{\sqrt{34}}i + \frac{5}{\sqrt{34}}j$$

a) $c\vec{v} = cai + cbj$

b) $v = \vec{u}\qquad if\ \ a_1 = a_2\ \&\ b_1 = b_2$

c) $v = 0\qquad if\ a = 0\ \ \&\ \ b = 0$