Examples:

1- *Find the direction and the length of* $\vec{A} = 3i - 4j$

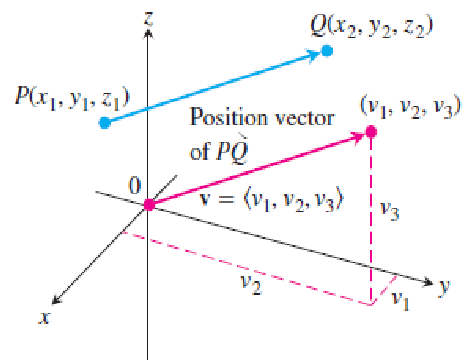
$$\text{Length of } \vec{A} = |\vec{A}| = \sqrt{3^2 + (-4)^2} = 5$$

$$\text{direction of } A = \frac{\vec{A}}{|\vec{A}|} = \frac{3i - 4j}{5} = \frac{3}{5}i - \frac{4}{5}j$$

$$= \cos\alpha i - \sin\alpha j$$

$$\therefore \cos\alpha = \frac{3}{5} \gg \alpha = 53°$$

Vector between two points:



Let P is the first point (x_1, y_1, z_1)

and Q is the second point (x_2, y_2, z_2)

$\overrightarrow{PQ} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$

OR $\overrightarrow{QP} = (x_1 - x_2)i + (y_1 - y_2)j + (z_1 - z_2)k$

$\therefore \overrightarrow{PQ} = -\overrightarrow{QP}$

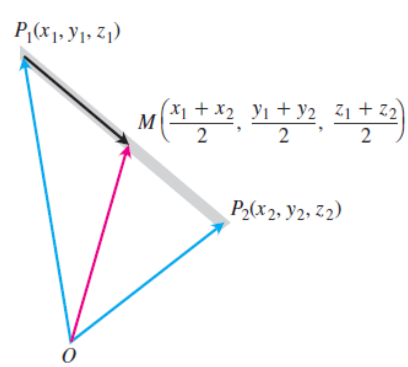
Ex.

1- *Find a vector joining A(2,5) & B(4,1)?*

$\overrightarrow{AB} = (4 - 2)i + (1 - 5)j = 2i - 4j$

$\overrightarrow{BA} = (2 - 4)i + (5 - 1)i = -2i + 4j$

Midpoint:



The midpoint of the line sigment joining points

$\overrightarrow{P_1}(x_1, y_1, z_1), \overrightarrow{P_2}(x_2, y_2, z_2)$ is the point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Ex. ***Find the midpoint of the sigment joining $P_1(3,-2,0)$&$P_2(7,4,4)$?***

$$\text{the midpoint is } \left(\frac{3+7}{2}, \frac{-2+4}{2}, \frac{0+4}{2}\right) = (5,1,2).$$

****The coordinates of a point which divides the join of points P_1 (x_1, y_1) and***

$P_2(x_2,y_2)$In the ratio m_1/m_2 are :

$$Q = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_2y_1 + m_1y_2}{m_1+m_2}\right)$$

Ex. ***Find the vector OP_3 where P_3 is the point which divide the line between $P_1(2,-1)$ $P_2(-4,3)$ with a ratio 3/2 ?***

$$P_3 = \left(\frac{3(-4)+2(2)}{3+2}, \frac{2(-1)+3(3)}{3+2}\right.$$

$$P_3 = \frac{-8}{5}i + \frac{7}{5}j$$

Notes:

1- *The vector* $ai + bj$ *is normal to the vector* $\mathbf{bi - aj}$.

2- *The vector* $ai + bj$ *is normal to the line* $ax + by = c$.

3- *The vector* $bi - aj$ *is the vector of the line* $ax + by = c$ $(parallel)$.

ملاحظة :

-1 المتجه $\mathbf{ai + bj}$ عمود على المتجه . $\mathbf{bi - aj}$

-2 المتجه $\mathbf{ai + bj}$ عمود عل المستقيم الذي معادلته . $\mathbf{ax + by = c}$

-3 المتجه $\mathbf{bi - aj}$ هو متجه المستفيم الذي معادلته $\mathbf{ax + by = c}$ (موازي له) .

Ex. *Find a vector that is normal to* $2i - 3j$?

$\vec{A} = 2i + 3j$

$\vec{N} = 3i - 2j$

Note: $\vec{A}$ *parallel* $\vec{B} \Leftrightarrow \vec{A} = t\vec{B}$

Where, t = *scaler* , $\vec{A}$ & $\vec{B}$ *are vectors*

Ex. *Show that* $\vec{A} = 2i + 3j$ & $\vec{B} = 4i + 6j$ *are parallel* .

$\vec{A} = 2i + 3j$

$\vec{B} = 4i + 6j = 2(2i + 3j) = 2\vec{A} \qquad \therefore \vec{A}$ *parallel* $\vec{B}$.

Ex. *Find a vector tangent and normal to the curve* $y = \frac{1}{3}x^3 + \frac{1}{2}$ *at point* $(1, 1)$.

Sol. $y = \frac{1}{3}x^3 + \frac{1}{2}$

$\bar{y} = x^2$

$at\ (1, 1) \rightarrow \bar{y} = 1$

$y = x + c \quad \rightarrow \quad y - x + c = 0$

$T = -i - j$

$N = i - j$

Ex. *Find a unit vector tangent and normal to the curve* $y = \frac{x^3}{2} + 5$ *at* $(1, 5.5)$.

Sol. $y = \frac{x^3}{2} + 5$

$at\ (1, 5.5)\ y = \frac{3}{2}$

$\therefore 3x - 2y = c$

$T = -2i - 3j$ & $N = 3i - 2j$

Unit vector $= \frac{\vec{T}}{|\vec{T}|} = \frac{-2i-3j}{\sqrt{(-2)^2+(-3)^2}} = \frac{-2}{\sqrt{20}} - \frac{3}{\sqrt{20}}$

Unit vector $= \frac{\vec{N}}{|\vec{N}|} = \frac{3i-2j}{\sqrt{(3)^2+(-2)^2}} = \frac{3}{\sqrt{20}} - \frac{2}{\sqrt{20}}$