1- $if\ \overrightarrow{v_1} \times \overrightarrow{v_2} \ \Leftrightarrow\ \overrightarrow{v_1}\ Parallel\ \overrightarrow{v_2}$

2- $if\ \ V = \vec{A} \times \vec{B}\ \ then\ \ |V| = area\ of\ parallelogram\,.$

Ex.: $if\ \ \overrightarrow{v_1} = 5i - j + k\ \ and\ \ \ v_2 = j - 5k$

$find\ \vec{C} = \overrightarrow{v_1} \times \overrightarrow{v_2}$

sol. $\vec{C} = \overrightarrow{v_1} \times \overrightarrow{v_2} = \begin{vmatrix} i & -j & k \\ 5 & -1 & 1 \\ 0 & 1 & -5 \end{vmatrix} = i[-1(-5) - 1(1)] - j[5(-5) - 0(-1)] +$ $k[5(1) - 0(-1)]$

$\vec{C} = 4i + 25j + 5k$

Triple Product (الضرب الثلاثي)

a) Scalar triple product

$\vec{A} \cdot (\vec{B} \times \vec{C})$ $\ scalar\ triple\ product$

$Let\,,\quad \vec{A} = a_1 i + b_1 j + c_1 k$

$\vec{B} = a_2 i + b_2 j + c_2 k$

$\vec{C} = a_3 i + b_3 j + c_3 k$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \overrightarrow{(A \times B)} \cdot \vec{C}$$

b) Vector triple product

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$$

Ex.: *Find a vector that is a normal to* $\vec{A} = 2i + 3j + k$ *and such that is lies in the plane determined by* $\vec{C} = i - j + 2k$, $\vec{B} = 2i - 4j - k$.

sol.

$L = N \times A$

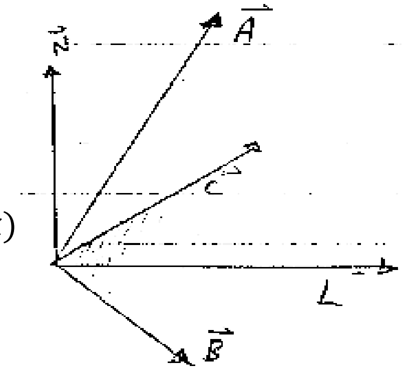
$L = \vec{A} \times (\vec{C} \times \vec{B})$

$L = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$

$L = (2 - 3 + 2)(2i - 4j - k) - (4 - 12 - 1)(i - j + 2k)$

$L = (2i - 4j - k) - 9(i - j + 2k)$

$L = 11i - 12j + 17k$

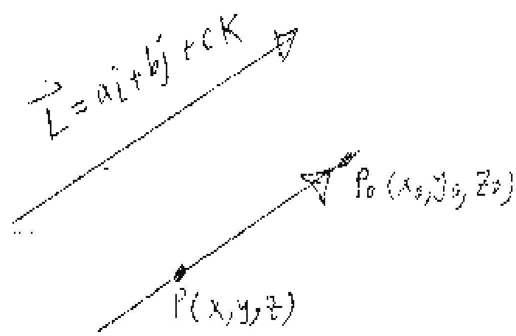


Equation of line in space

$$\overrightarrow{PP_o} = (x - x_o)i + (y - y_o)j + (z - z_o)k$$

$$if\ \overrightarrow{PP_o}\ parallel\ \vec{L}$$

$$(x - x_o)i + (y - y_o)j + (z - z_o)k = t(ai + bj + ck)$$



There are two forms of equation of line in space:

1- <u>Parametric Equation</u>

$x = x_o + at$

$y = y_o + bt$

$z = z_o + ct$

Where, x_o, y_o, z_o *is point on the line*

And x,y,z is parametric of parallel vector

2- *Standard form (Cartesian form)*

$$\frac{x-x_o}{a}=\frac{y-y_o}{b}=\frac{z-z_o}{c}=t \qquad a\neq 0,\ b\neq 0,\ c\neq 0$$

Ex. *Find the equation of the line that passes through A(2,1,4) and B(3,2,7)*

Sol.

$\overrightarrow{AB}=i+j+3k$

$x_o=2,\quad y_o=1,\quad z_o=4$

$$\frac{x-2}{1}=\frac{y-1}{1}=\frac{z-4}{3} \qquad \textit{standard form}$$

$x=2+t$

$y=1+t$ *parametric form*

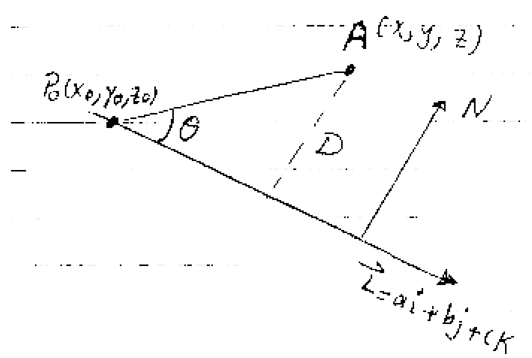
$x=4+3t$

Note: *to find equation of line in space we need :*

1- Point on the line

2- Vector parallel to the line

Distance from a point to a line



There are to method to find the distance from a point to a line :