

Multi-Dimensional Unsteady-state Heat Conduction

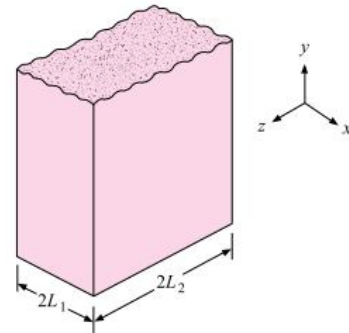
The Heisler charts are used to obtain the temperature distribution in the infinite plate of thickness $2L$, in the long cylinder, or in the sphere. For the problems in which heat transferred with more than one dimensional, it is possible to combine the solutions for one-dimensional systems obtain the solutions for the multidimensional problems.

For the rectangular bar,

$$\left(\frac{T - T_{\infty}}{T_i - T_{\infty}} \right)_{\text{bar}} = \left(\frac{T - T_{\infty}}{T_i - T_{\infty}} \right)_{2L_1 \text{ plate}} \left(\frac{T - T_{\infty}}{T_i - T_{\infty}} \right)_{2L_2 \text{ plate}}$$

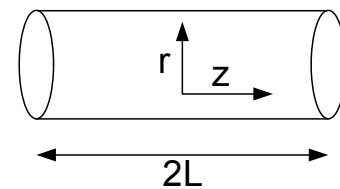
or

$$\left(\frac{\theta}{\theta_i} \right)_{\text{bar}} = \left(\frac{\theta}{\theta_i} \right)_{2L_1} \left(\frac{\theta}{\theta_i} \right)_{2L_2}$$



For short cylinder r,

$$\left(\frac{\theta}{\theta_i} \right)_{\text{cylM}} = \left(\frac{\theta}{\theta_i} \right)_{\text{inf inite cyl.}} \left(\frac{\theta}{\theta_i} \right)_{\text{inf inite plate}}$$



if $C(\theta)$ = solution for infinite cylinder

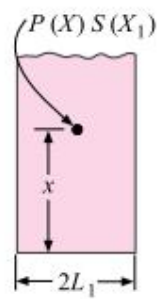
$P(x)$ = solution for infinite plate

$S(x)$ = solution for semi - infinite solid

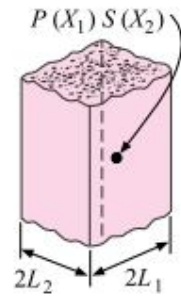
The general is

$$\left(\frac{\theta}{\theta_i} \right)_{\text{combined solid}} = \left(\frac{\theta}{\theta_i} \right)_{\text{intersection solid 1}} \left(\frac{\theta}{\theta_i} \right)_{\text{intersection solid 2}} \left(\frac{\theta}{\theta_i} \right)_{\text{intersection solid 3}}$$

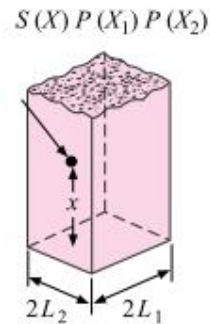
Figure 4-18 | Product solutions for temperatures in multidimensional systems:
 (a) semi-infinite plate;
 (b) infinite rectangular bar;
 (c) semi-infinite rectangular bar;
 (d) rectangular parallelepiped;
 (e) semi-infinite cylinder;
 (f) short cylinder.



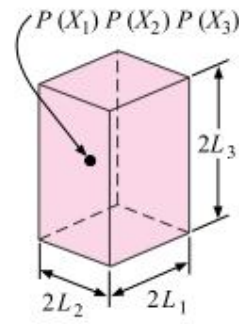
(a)



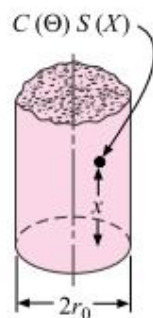
(b)



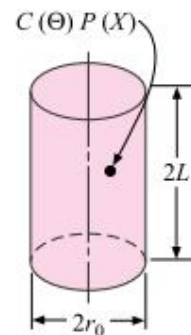
(c)



(d)



(e)



(f)

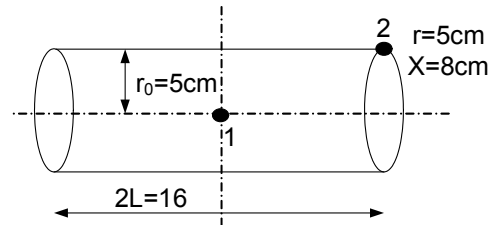
Example: A 10cm diameter, 16cm long cylinder with properties: $k=0.5 \text{ W/m.K}$ and $\alpha=5 \times 10^{-7} \text{ m}^2/\text{s}$ is initially at a temperature of 20°C . The cylinder is placed in an oven where the ambient temperature is 500°C , $h = 30 \text{ W/m}^2.\text{K}$. Determine the min and max temperature in the cylinder 30min after it has been placed in the oven.

Solution

$$2L = 16 \Rightarrow L = x = 8\text{cm}, \quad t = 30 \times 60 = 1800\text{sec}$$

min temperature at $x = 0, \quad r = 0$

max temperature at $x = L, \quad r = r_0$



1) infinite plate

$$\frac{\alpha t}{L^2} = 0.14, \quad \frac{k}{hL} = 0.21$$

$$\text{From figure 4.9 } \frac{\theta_0}{\theta_i} = 0.9$$

$$\text{From figure 4.12, } \frac{\theta}{\theta_0} = 0.27$$

$$\frac{\theta}{\theta_i} = \frac{\theta}{\theta_0} \cdot \frac{\theta_0}{\theta_i} = 0.27 \times 0.9 = 0.249$$

2) infinite cylinder

$$\frac{k}{hr_0} = 0.33, \quad \frac{\alpha t}{r_0^2} = 0.36$$

$$\text{From figure 4.10 } \frac{\theta_0}{\theta_i} = 0.47$$

$$\text{From figure 4.13, } \frac{\theta}{\theta_0} = 0.33$$

$$\frac{\theta}{\theta_i} = \frac{\theta}{\theta_0} \cdot \frac{\theta_0}{\theta_i} = 0.33 \times 0.47 = 0.155$$

$$\text{The min temperature} = \left. \frac{\theta_0}{\theta_i} \right|_{\text{combined}} = \left. \frac{\theta_0}{\theta_i} \right|_{\text{plate}} \cdot \left. \frac{\theta_0}{\theta_i} \right|_{\text{cylinder}} = 0.9 \times 0.47 = 0.423$$

$$\therefore T_{\min} = 500 + 0.423(20 - 500) = 297^\circ\text{C}$$

$$\text{The max temperature} = \left. \frac{\theta}{\theta_i} \right|_{\text{combined}} = \left. \frac{\theta}{\theta_i} \right|_{\text{plate}} \cdot \left. \frac{\theta}{\theta_i} \right|_{\text{cylinder}} = 0.249 \times 0.155 = 0.039$$

$$\therefore T_{\max} = 500 + 0.039(20 - 500) = 481^\circ\text{C}$$