Principles of Convection

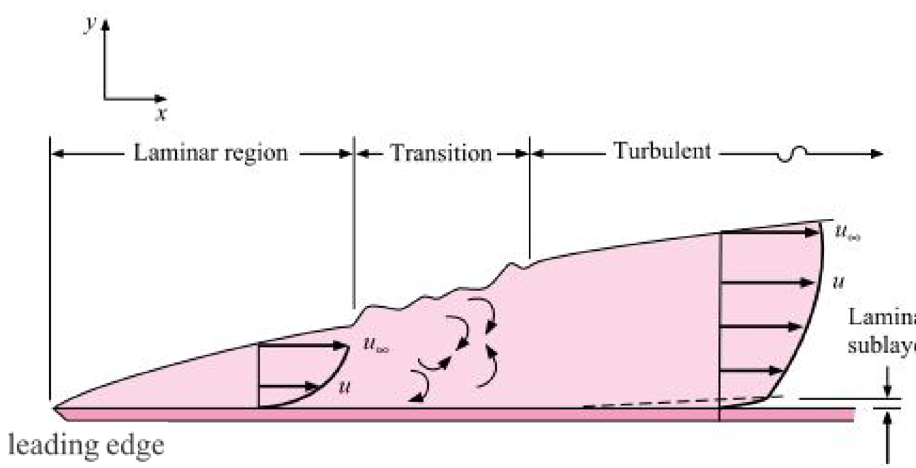
Convection was considered only insofar as it related to the boundary conditions imposed on a conduction problem. The steady of convection will be focused on the methods of calculating convection heat transfer specially, the ways of predicting the value of the convection heat-transfer coefficient *h* (depends on the density, viscosity and fluid velocity in addition to its thermal properties) that requires an energy balance along with an analysis of the fluid dynamics of the problems concerned.

To steady the convection, we may discuss:

1- Fluid dynamics and boundary-layer analysis.
2- Energy balance on the flow system and determine the influence of the flow on the temperature gradients in the fluid.
3- The heat-transfer rate from a heated surface to a fluid.

Viscous Flow

Figure 5-1 | Sketch showing different boundary-layer flow regimes on a flat plate.



- For a flow over a flat plate as shown in Figure, different flow region develops by the influence of viscous forces.
- The viscous forces are described in terms of a shear stress τ between the fluid layers.
- The region of flow that develops from the leading edge of the plate in which the effects of viscosity are observed is called the boundary layer.

There are three stages

1- Laminar, flow of the fluid is laminar.
2- Transition, small disturbances in flow.
3- Turbulent, random flow which is characterized by eddies.

For flat plate the transition from laminar to turbulent flow occurs when

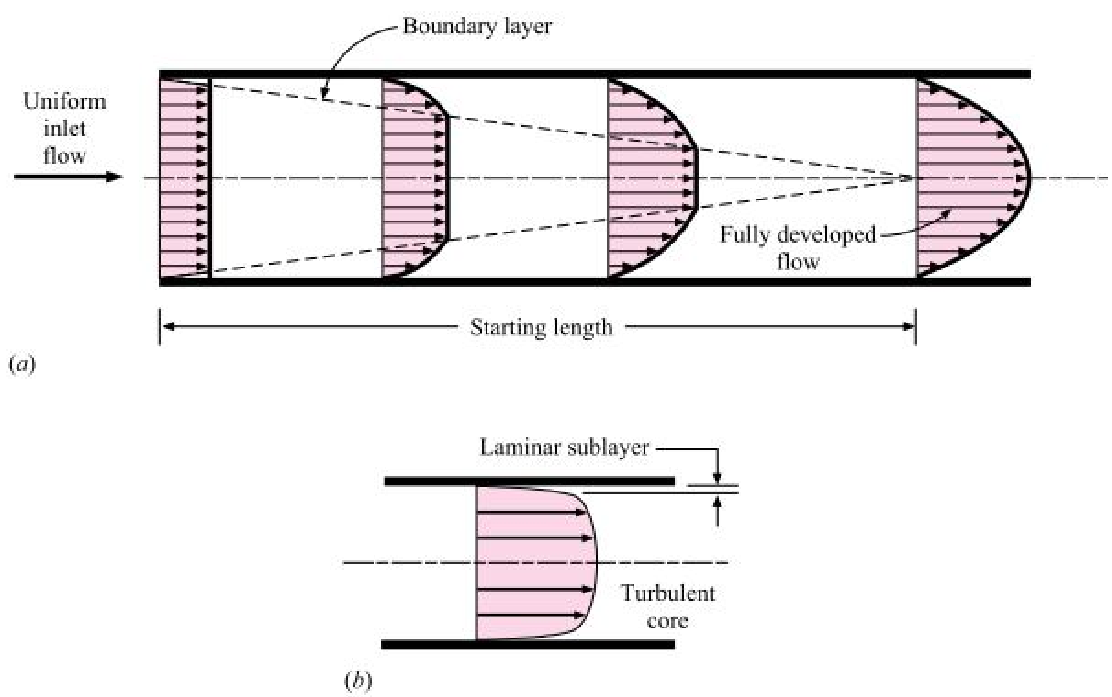
$$\frac{u_\infty x}{\nu} = \frac{\rho u_\infty x}{\mu} > 5 \times 10^5$$

Where
u_∞ = free-stream velocity, m/s
x = distance from leading edge, m
$\nu = \mu/\rho$ = kinematic viscosity, m^2/s

As shown in the figure, the velocity profile in laminar section is approximately parabolic, while the turbulent profile has a portion near the wall that is very nearly linear. This linear portion is called a *laminar sublayer*.

Flow in Pipes

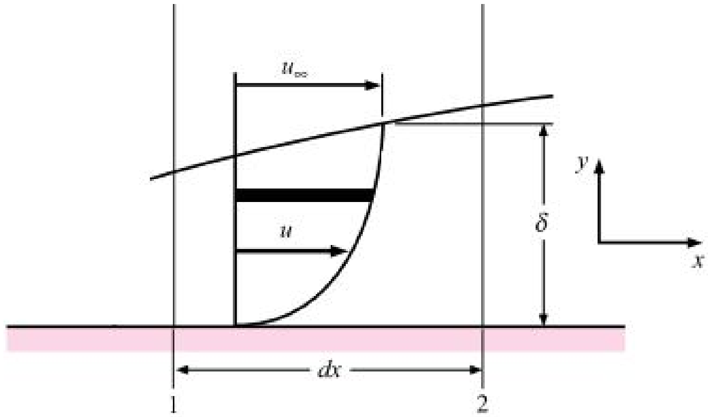
Figure 5-3 | Velocity profile for (*a*) laminar flow in a tube and (*b*) turbulent tube flow.



A boundary layer develops at the entrance, as shown. Eventually the boundary layer fills the entire tube, and the flow is said to be fully developed. If the flow is laminar, a parabolic velocity profile is noted. When the flow is turbulent, a blunter profile is observed.

$$\mathrm{Re} = \frac{\rho\, u\, d}{\mu} \langle\ 2300 \textit{ for laminar flow}$$

Laminar Boundary Layer on a Flat Plate



Consider the boundary-layer flow system shown in Figure. The free-stream velocity outside the boundary layer is u_∞ , and the boundary-layer thickness is δ.

The thickness of the boundary layer at any distance on the flat plate can be estimated by using the following equation

$$\frac{\delta}{x} = \frac{4.64}{\mathrm{Re}_x^{1/2}}$$

where

$$\mathrm{Re}_x = \frac{u_\infty x}{\nu}$$

Mass Flow and Boundary-Layer Thickness — EXAMPLE 5-3

Air at 27°C and 1 atm flows over a flat plate at a speed of 2 m/s. Calculate the boundary-layer thickness at distances of 20 cm and 40 cm from the leading edge of the plate. The viscosity of air at 27°C is 1.85×10^{-5} kg/m · s.

Solution

The density of air is calculated from

$$\rho = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(300)} = 1.177 \text{ kg/m}^3 \quad [0.073 \text{ lb}_m/\text{ft}^3]$$

The Reynolds number is calculated as

$$\text{At } x = 20 \text{ cm:} \qquad \mathrm{Re} = \frac{(1.177)(2.0)(0.2)}{1.85 \times 10^{-5}} = 25{,}448$$

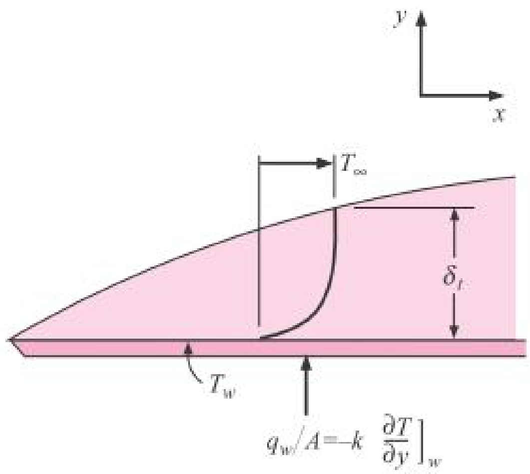
$$\text{At } x = 40 \text{ cm:} \qquad \mathrm{Re} = \frac{(1.177)(2.0)(0.4)}{1.85 \times 10^{-5}} = 50{,}897$$

At $x = 20$ cm: $\delta = \dfrac{(4.64)(0.2)}{(25{,}448)^{1/2}} = 0.00582$ m [0.24 in]

At $x = 40$ cm: $\delta = \dfrac{(4.64)(0.4)}{(50{,}897)^{1/2}} = 0.00823$ m [0.4 in]

The thermal Boundary Layer

Figure 5-7 | Temperature profile in the thermal boundary layer.



A thermal boundary layer is defined as that region where temperature gradients are present in the flow. These temperature gradients would result from a heat-exchange process between the fluid and the wall.
The temperature of the wall is Tw, the temperature of the fluid outside the thermal boundary layer is $T\infty$, and the thickness of the thermal boundary layer is designated as δt. At the wall, the velocity is zero, and the heat transfer into the fluid takes place by conduction. Thus the local heat flux per unit area, q'' , is

$$\frac{q}{A} = q'' = -k \left.\frac{\partial T}{\partial y}\right]_{\text{wall}}$$

From Newton's law of cooling

$$q'' = h(T_w - T_\infty)$$

Where h is the convection heat-transfer coefficient. Combining these equations, we have

$$h = \frac{-k(\partial T/\partial y)_{\text{wall}}}{T_w - T_\infty}$$

$\left.\frac{\partial T}{\partial y}\right]_{\text{wall}}$: is the temperature gradient at the wall thickness of thermal boundary layer.

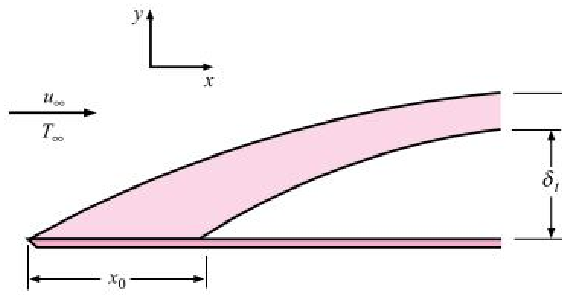
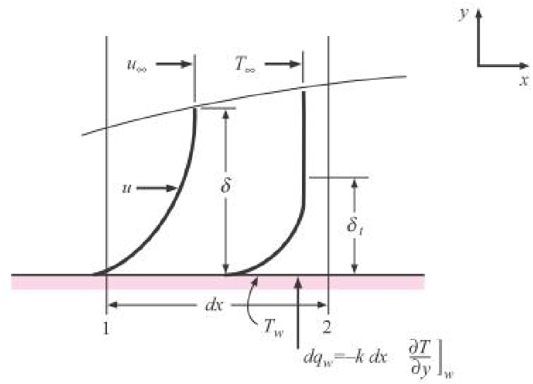


Figure 5-8 | Control volume for integral energy analysis of laminar boundary flow.



The thickness boundary layer can be calculated by using the following equation

$$\frac{\delta_t}{\delta} = \frac{1}{1.026}\mathrm{Pr}^{-1/3}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}$$

Where
Pr = Prandtl number

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho c_p} = \frac{c_p\mu}{k}$$

x_0= is the distance from the leading edge where the heating begins.

When the plate is heated over the entire length, $x_0 = 0$, then

$$\frac{\delta_t}{\delta} = \frac{1}{1.026}\mathrm{Pr}^{-1/3}$$

Heat Transfer Coefficient

The heat transfer coefficient at any x position on the flat plate can be found by the equation

$$h_x = 0.332k\,\mathrm{Pr}^{1/3}\left(\frac{u_\infty}{\nu x}\right)^{1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

The last equation can be written in dimensionless form as

$$\mathrm{Nu}_x = 0.332\mathrm{Pr}^{1/3}\ \mathrm{Re}_x^{1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

For a plate heated over its entire length, $x_0 = 0$ and

$$\mathrm{Nu}_x = 0.332\mathrm{Pr}^{1/3}\ \mathrm{Re}_x^{1/2}$$

Where

Nu_x= Nusselt number = $\dfrac{h_x\, x}{k}$

The above equation expresses the local values of the heat-transfer coefficient in terms of the distance from the leading edge of the plate and the fluid properties. For the case where x0 = 0 the average heat-transfer coefficient and Nusselt number may be obtained by integrating over the length of the plate:

$$\bar{h} = \frac{\int_0^L h_x\, dx}{\int_0^L dx} = 2h_{x=L}$$

Where

$\bar{h}$ is the average heat transfer coefficient.

$\therefore$

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = 2\ \mathrm{Nu}_{x=L}$$

or

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = 0.664\ \mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3}$$

where

$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu}$$

The above relations are used for laminar flow constant wall temperature. Note that, all the physical properties are evaluated at film temperature T_f, which is

$$T_f = \frac{T_w + T_\infty}{2}$$