Constant Heat Flux

The above analysis has considered the laminar heat transfer from an isothermal surface. For the constant-heat-flux case) the objective is to find the distribution of the plate-surface temperature), it can be shown that the local Nusselt number is given by

$$\mathrm{Nu}_x = \frac{hx}{k} = 0.453\ \mathrm{Re}_x^{1/2}\ \mathrm{Pr}^{1/3}$$

$$\frac{q}{A} = q_w = h(T_W - T_\infty)$$

The average value of $Tw\text{-}T\infty$ ($\overline{T_w - T_\infty}$ can be calculated by integration over the whole plate (for $x=0$ to $x=L$)

$$\overline{T_w - T_\infty} = \frac{1}{L}\int_0^L (T_w - T_\infty)\,dx = \frac{1}{L}\int_0^L \frac{q_w x}{k\,\mathrm{Nu}_x}\,dx$$
$$= \frac{q_w L/k}{0.6795\ \mathrm{Re}_L^{1/2}\ \mathrm{Pr}^{1/3}}$$

Note that, all the physical properties for constant heat flux are evaluated at film temperature T_f, which is

$$T_f = \frac{\overline{T_W + T_\infty} + T_\infty}{2}$$

All previous relations are applicable for Prandtl numbers between about 0.6 and 50. for values of Pr<0.6 or Pr>50, the following equation is used for constant wall temperature

$$\mathrm{Nu}_x = \frac{0.3387\ \mathrm{Re}_x^{1/2}\ \mathrm{Pr}^{1/3}}{\left[1 + \left(\dfrac{0.0468}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}} \qquad \text{for } \mathrm{Re}_x\,\mathrm{Pr} > 100$$

In the above equation, for the **constant-heat-flux** case, 0.3387 is changed to 0.4637 and 0.0468 is changed to 0.0207. Properties are still evaluated at the film temperature.

Example 5.4 in Holman.

EXAMPLE 5-5 Flat Plate with Constant Heat Flux

A 1.0-kW heater is constructed of a glass plate with an electrically conducting film that produces a constant heat flux. The plate is 60 cm by 60 cm and placed in an airstream at 27°C, 1 atm with $u_\infty = 5$ m/s. Calculate the average temperature difference along the plate and the temperature difference at the trailing edge.

■ Solution

Properties should be evaluated at the film temperature, but we do not know the plate temperature. So for an initial calculation, we take the properties at the free-stream conditions of

$$T_\infty = 27°\text{C} = 300\ \text{K}$$

$$\nu = 15.69 \times 10^{-6}\ \text{m}^2/\text{s} \qquad \text{Pr} = 0.708 \qquad k = 0.02624\ \text{W/m}\cdot°\text{C}$$

$$\text{Re}_L = \frac{(0.6)(5)}{15.69 \times 10^{-6}} = 1.91 \times 10^5$$

From Equation (5-50) the average temperature difference is

$$\overline{T_w - T_\infty} = \frac{[1000/(0.6)^2](0.6)/0.02624}{0.6795(1.91 \times 10^5)^{1/2}(0.708)^{1/3}} = 240°\text{C}$$

Now, we go back and evaluate properties at

$$T_f = \frac{240 + 27 + 27}{2} = 147°\text{C} = 420\ \text{K}$$

and obtain

$$\nu = 28.22 \times 10^{-6}\ \text{m}^2/\text{s} \qquad \text{Pr} = 0.687 \qquad k = 0.035\ \text{W/m}\cdot°\text{C}$$

$$\text{Re}_L = \frac{(0.6)(5)}{28.22 \times 10^{-6}} = 1.06 \times 10^5$$

$$\overline{T_w - T_\infty} = \frac{[1000/(0.6)^2](0.6)/0.035}{0.6795(1.06 \times 10^5)^{1/2}(0.687)^{1/3}} = 243°\text{C}$$

At the end of the plate ($x = L = 0.6$ m) the temperature difference is obtained from Equations (5-48) and (5-50) with the constant 0.453 to give

$$(T_w - T_\infty)_{x=L} = \frac{(243.6)(0.6795)}{0.453} = 365.4°\text{C}$$

An alternate solution would be to base the Nusselt number on Equation (5-51).

Plate with Unheated Starting Length EXAMPLE 5-6

Air at 1 atm and 300 K flows across a 20-cm-square plate at a free-stream velocity of 20 m/s. The last half of the plate is heated to a constant temperature of 350 K. Calculate the heat lost by the plate.

■ Solution

First we evaluate the air properties at the film temperature

$$T_f = (T_w + T_\infty)/2 = 325\ \text{K}$$

and obtain

$$\upsilon = 18.23 \times 10^{-6} \mathrm{m}^2/\mathrm{s} \quad k = 0.02814 \ \mathrm{W/m \cdot °C} \quad \mathrm{Pr} = 0.7$$

At the trailing edge of the plate the Reynolds number is

$$\mathrm{Re}_L = u_\infty L/\upsilon = (20)(0.2)/18.23 \times 10^{-6} = 2.194 \times 10^5$$

or, laminar flow over the length of the plate.

Heating does not start until the last half of the plate, or at a position $x_0 = 0.1$ m. The local heat-transfer coefficient for this condition is given by Equation (5-41):

$$h_x = 0.332k \,\mathrm{Pr}^{1/3}(u_\infty/\upsilon x)^{1/2}[1 - (x_0/x)^{0.75}]^{-1/3} \qquad \textbf{[a]}$$

Inserting the property values along with $x_0 = 0.1$ gives

$$h_x = 8.6883x^{-1/2}(1 - 0.17783x^{-0.75})^{-1/3} \qquad \textbf{[b]}$$

The plate is 0.2 m wide so the heat transfer is obtained by integrating over the heated length $x_0 < x < L$

$$q = (0.2)(T_w - T_\infty)\int_{x_0=0.1}^{L=0.2} h_x dx \qquad \textbf{[c]}$$

Inserting Equation (b) in Equation (c) and performing the numerical integration gives

$$q = (0.2)(8.6883)(0.4845)(350 - 300) = 421 \ \mathrm{W} \qquad \textbf{[d]}$$

The average value of the heat-transfer coefficient *over the heated length* is given by

$$h = q/(T_w - T_\infty)(L - x_0)W = 421/(350 - 300)(0.2 - 0.1)(0.2) = 421 \ \mathrm{W/m^2 \cdot °C}$$

where W is the width of the plate.

An easier calculation can be made by applying Equation (5-45b) to determine the average heat transfer coefficient over the heated portion of the plate. The result is

$$h = 425.66 \ \mathrm{W/m^2 \cdot °C} \quad \text{and} \quad q = 425.66 \ \mathrm{W}$$

which indicates, of course, only a small error in the numerical integeration.

EXAMPLE 5-7 Oil Flow Over Heated Flat Plate

Engine oil at 20°C is forced over a 20-cm-square plate at a velocity of 1.2 m/s. The plate is heated to a uniform temperature of 60°C. Calculate the heat lost by the plate.

■ Solution

We first evaluate the film temperature:

$$T_f = \frac{20 + 60}{2} = 40°\mathrm{C}$$

The properties of engine oil are

$$\rho = 876\ \text{kg/m}^3 \qquad \nu = 0.00024\ \text{m}^2/\text{s}$$
$$k = 0.144\ \text{W/m}\cdot{}^\circ\text{C} \qquad \text{Pr} = 2870$$

The Reynolds number is

$$\text{Re} = \frac{u_\infty L}{\nu} = \frac{(1.2)(0.2)}{0.00024} = 1000$$

Because the Prandtl number is so large we will employ Equation (5-51) for the solution. We see that h_x varies with x in the same fashion as in Equation (5-44), that is, $h_x \propto x^{-1/2}$, so that we get the same solution as in Equation (5-45) for the average heat-transfer coefficient. Evaluating Equation (5-51) at $x = 0.2$ gives

$$\text{Nu}_x = \frac{(0.3387)(1000)^{1/2}(2870)^{1/3}}{\left[1 + \left(\dfrac{0.0468}{2870}\right)^{2/3}\right]^{1/4}} = 152.2$$

and

$$h_x = \frac{(152.2)(0.144)}{0.2} = 109.6\ \text{W/m}^2\cdot{}^\circ\text{C}$$

The average value of the convection coefficient is

$$h = (2)(109.6) = 219.2\ \text{W/m}^2\cdot{}^\circ\text{C}$$

so that the total heat transfer is

$$q = hA(T_w - T_\infty) = (219.2)(0.2)^2(60 - 20) = 350.6\ \text{W}$$