The Relation Between Fluid Friction and Heat Transfer

Since the heat and momentum flow are related, we may obtain a relationship that relate the frictional resistance to the heat transfer. This analogy can be written as:

$$St\,Pr^{2/3} = Jf = 0.332Re^{-0.5}$$

This relation is called the Reynolds-Colburn analogy, expresses the relation between fluid friction and heat transfer for laminar flow on a flat plate.

Where:

St = Stanton number = $\dfrac{Nu}{\mathrm{Re}\,Pr} = \dfrac{h}{\rho\,cp\,u_\infty} = 0.332\ Pr^{-2/3}\ Re^{-1/2}$

Jf = friction factor

Example: Air at 20 kPa and 20°C flow across flat plate 0.6m long. The free stream velocity is 30m/s. the plate is heated over its entire length to a temperature of 55°C, a)for x=0.3m, calculate the value of y for which u will equal 22.5m/s b)estimate the value of fiction factor at distance 0.15m from the leading edge. Note: the velocity distribution as

$$\frac{u}{u_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Solution:

$$T_f = \frac{20+55}{2} = 37.5°C,\ u_\infty = 30m/s$$

$$\rho_{air} = \frac{P.Mw}{R.T} = 0.224kg/m^3$$

$$\mu = 2\times10^{-5}\ pa.s$$

$$a)\ at\ x = 0.3m, Re_x = \frac{\rho\,u\,x}{\mu} = 100800$$

$$\frac{\delta}{x} = 4.64\,R_x^{-1/2} \Rightarrow \delta = 0.00438m$$

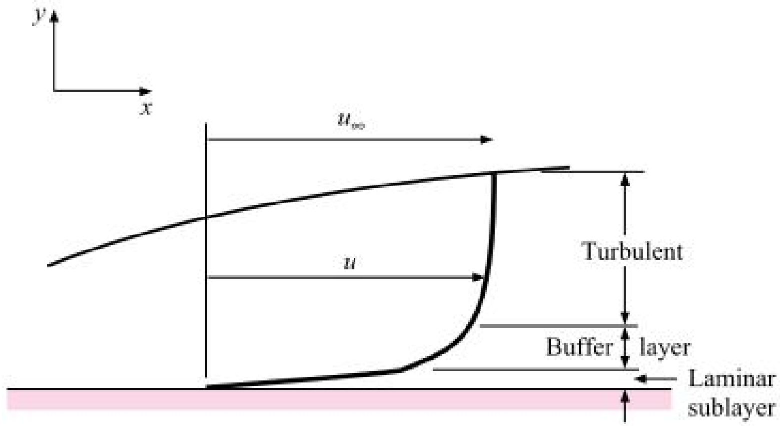
$$\frac{22.5}{30} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \Rightarrow \frac{y}{\delta} = 0.56 \Rightarrow y = 0.00245m$$

$$b)\ J_f = 0.332\,\mathrm{Re}_x^{-1/2}$$

$$at\ x = 0.15m \Rightarrow Re_x = 50400$$

$$\therefore\ J_f = 1.47\times10^{-3}$$

Turbulent Boundary Layer Heat Transfer



Three region can be observed:

1) A very thin region near the plate surface is called "the laminar sublayer" has the same character of laminar flow.
2) At larger y distances from the plate, a layer is called "buffer layer" in which some turbulent action is found in addition to the molecular viscous action and heat conduction.
3) Turbulent layer, in which the flow is fully turbulent.

Turbulent Heat Transfer Based on Fluid-Friction Analogy

The friction factor in turbulent flow on flat plate can be calculated by

$$J_f = 0.0592\, Re_x^{-1/5} \quad for \quad 5\times 10^5 \langle \mathrm{Re}_x \langle 10^7$$

$$J_f = 0.370\, (\log Re_x)^{-2.584} \quad for \quad 10^7 \langle \mathrm{Re}_x \langle 10^9$$

The average-friction coefficient for a flat plate with a laminar boundary layer and turbulent thereafter can be calculated from

$$\overline{J_f} = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1700}{\mathrm{Re}_L} \qquad for\ Re_L \langle 10^7$$

$$\overline{J_f} = \frac{0.455}{(\log \mathrm{Re}_L)^{2.584}} - \frac{1700}{\mathrm{Re}_L} \qquad for\ Re_L \langle 10^9$$

Applying the fluid-friction analogy St $Pr^{2/3} = J_f$ we obtain the local turbulent heat transfer as:

$$St_x \, Pr^{2/3} = 0.0296 \, Re_x^{-1/5} \qquad 5 \times 10^5 < Re_x < 10^7$$

$$St_x \, Pr^{2/3} = 0.185 (\log Re_x)^{-2.584} \qquad 10^7 < Re_x < 10^9$$

The average heat transfer over the entire laminar-turbulent boundary layer is

$$\overline{St} \, Pr^{2/3} = 0.037 \, Re_L^{-1/5} - 871 \, Re_L^{-1} \qquad for \quad 5 \times 10^5 \langle Re_L \langle 10^7$$

Since $St = Nu / Re.Pr$ $\Rightarrow$

$$\overline{Nu}_L = \frac{\bar{h} L}{k} = Pr^{1/3} (0.037 \, Re_L^{0.8} - 871)$$

Constant Heat Flux

For constant-wall-heat flux in turbulent flow, the local Nusselt numbers is only about 4 % higher than for the isothermal surface; that is,

$$Nu_x = 1.04 \, Nu_x \Big]_{T_w = \text{const}}$$

Turbulent Heat Transfer from Isothermal Flat Plate

EXAMPLE 5-9

Air at 20°C and 1 atm flows over a flat plate at 35 m/s. The plate is 75 cm long and is maintained at 60°C. Assuming unit depth in the z direction, calculate the heat transfer from the plate.

Solution

We evaluate properties at the film temperature:

$$T_f = \frac{20 + 60}{2} = 40°C = 313 \text{ K}$$

$$\rho = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(313)} = 1.128 \text{ kg/m}^3$$

$$\mu = 1.906 \times 10^{-5} \text{ kg/m} \cdot \text{s}$$

$$Pr = 0.7 \qquad k = 0.02723 \text{ W/m} \cdot °C \qquad c_p = 1.007 \text{ kJ/kg} \cdot °C$$

The Reynolds number is

$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu} = \frac{(1.128)(35)(0.75)}{1.906 \times 10^{-5}} = 1.553 \times 10^6$$

and the boundary layer is turbulent because the Reynolds number is greater than 5×10^5. Therefore, we use Equation (5-85) to calculate the average heat transfer over the plate:

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = \mathrm{Pr}^{1/3}(0.037\,\mathrm{Re}_L^{0.8} - 871)$$

$$= (0.7)^{1/3}[(0.037)(1.553 \times 10^6)^{0.8} - 871] = 2180$$

$$\bar{h} = \overline{\mathrm{Nu}}_L \frac{k}{L} = \frac{(2180)(0.02723)}{0.75} = 79.1\ \mathrm{W/m^2 \cdot {}^\circ C} \quad [13.9\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}]$$

$$q = \bar{h}A(T_w - T_\infty) = (79.1)(0.75)(60 - 20) = 2373\ \mathrm{W} \quad [8150\ \mathrm{Btu/h}]$$

Turbulent Boundary Layer Thickness

The velocity profile in a turbulent boundary layer, outside the laminar sublayer, can be described as

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

For the case on the boundary layer is fully turbulent from the leading edge of the plate with the condition that $\delta = 0$ at $x = 0$ to, the boundary layer thickness is calculated

$$\frac{\delta}{x} = 0.381\,\mathrm{Re}_x^{-1/5}$$

For the boundary layer follows a laminar growth pattern up to $Re_{crit} = 5 \times 10^5$ and a turbulent growth thereafter

$$\frac{\delta}{x} = 0.381\,\mathrm{Re}_x^{-1/5} - 10{,}256\,\mathrm{Re}_x^{-1} \qquad 5 \times 10^5 < \mathrm{Re}_x < 10^7$$

Example 5.10 for this purpose.