Heat Transfer In Laminar Tube Flow

For laminar tube flow, the parabolic velocity distribution exists as shown. The velocity distribution can be represented

\[ \frac{u}{u_0} = 1 - \frac{r^2}{r_o^2} \]

Where \( r_o \) is the radius of the tube, and \( u_0 \) the velocity at the center of the tube.

The temperature distribution in the laminar tube flow may be written in terms of the temperature at the center of the tube (\( T_c \)) for constant heat flux.

\[ T - T_c = \frac{1}{r_o} \frac{\partial T}{\partial x} \left( \frac{u_0 r_o^2}{4} \right)^2 \left[ \left( \frac{r}{r_o} \right)^2 - \frac{1}{4} \left( \frac{r}{r_o} \right)^4 \right] \]

The gradient \( \partial T/\partial x \) is constant for the case of constant heat flux.

The Bulk Temperature

In tube flow, the convection heat-transfer coefficient is usually defined by

\[ q/A = h(T_w - T_b) \]

Where \( T_w \) is the wall temperature and \( T_b \) the so-called bulk temperature.

The bulk temperature is defined as the energy-average fluid temperature across the tube, which may be calculated from

\[ T_b = \bar{T} = \frac{\int_0^{r_o} \rho 2\pi r \, dr \, u c_p \, T}{\int_0^{r_o} \rho 2\pi r \, dr \, u c_p} \]

The numerator represents the total energy flow through the tube, and the denominator represents the product of mass flow and specific heat integrated over the flow area.

Substituting the temperature (\( T=T_c \)), and velocity distribution for laminar tube flow (\( u=u_0 \)) in the last equation we can get an expression for bulk temperature after integration.
Heat Transfer

Third Year

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\[ T_b = T_c + \frac{7}{96} \frac{u_0 r_o^2}{\alpha} \frac{\partial T}{\partial x} \]  

[5-103]

and for the wall temperature

\[ T_w = T_c + \frac{3}{16} \frac{u_0 r_o^2}{\alpha} \frac{\partial T}{\partial x} \]  

[5-104]

The heat-transfer coefficient is calculated from

\[ q = h A (T_w - T_b) = k A \left( \frac{\partial T}{\partial r} \right)_{r = r_o} \]  

[5-105]

\[ h = \frac{k (\partial T/\partial r)_{r = r_o}}{T_w - T_b} \]

The temperature gradient is given by

\[ \frac{\partial T}{\partial r} \bigg|_{r = r_o} = \frac{u_0}{\alpha} \frac{\partial T}{\partial x} \left( \frac{r}{2} - \frac{r^3}{4 r_o^2} \right)_{r = r_o} = \frac{u_0 r_o}{4 \alpha} \frac{\partial T}{\partial x} \]  

[5-106]

Substituting Equations (5-103), (5-104), and (5-106) in Equation (5-105) gives

\[ h = \frac{24}{11} \frac{k}{r_o} = \frac{48}{11} \frac{k}{d_o} \quad \text{for constant heat flux} \quad \text{inside tube} \]

Expressed in terms of the Nusselt number, the result is

\[ \text{Nu}_{d} = \frac{hd_o}{k} = 4.364 \]  

[5-107]

This means that \( \text{Nu} = 4.364 \) (constant) for laminar flow inside tube for constant heat flux.

Empirical and Practical Relations for Forced-Convection Heat Transfer

1- For fully developed turbulent flow in smooth tubes with moderate temperature difference

\[ \text{Nu}_{d} = 0.023 \text{Re}^{0.8}_{d} \text{Pr}^{n} \]

\[ n = \begin{cases} 0.4 & \text{for heating of the fluid} \\ 0.3 & \text{for cooling of the fluid} \end{cases} \]

Conditions: fully developed turbulent flow, smooth tubes for fluids, Prandtl numbers from 0.6 to 100, and with moderate temperature differences between wall and fluid conditions.

2- For wide temperature differences, turbulent flow in smooth tubes
\[
\text{Nu}_d = 0.027 \text{Re}_d^{0.8} \text{Pr}^{1/3} \left( \frac{\mu}{\mu_w} \right)^{0.14}
\]

All properties are evaluated at bulk-temperature conditions, except \( \mu_w \) (viscosity), which is evaluated at the wall temperature.

3- For entrance region and the flow is not developed

\[
\text{Nu}_d = 0.036 \text{Re}_d^{0.8} \text{Pr}^{1/3} \left( \frac{d}{L} \right)^{0.055} \quad \text{for} \quad 10 < \frac{L}{d} < 400
\]

Where \( L \) is the length of the tube and \( d \) is the tube diameter. The properties are evaluated at the mean bulk temperature.

4- For laminar flow (fully developed) in tubes at constant wall temperature

\[
\overline{\text{Nu}}_d = 3.66 + \frac{0.0668 (d/L) \text{Re}_d \text{Pr}}{1 + 0.04 [(d/L) \text{Re}_d \text{Pr}]^{2/3}}
\]

For very long tube \( \text{Nu}_d = 3.66 \)

5- Another relation is used for laminar flow inside smooth tubes (fully developed)

\[
\overline{\text{Nu}}_d = 1.86 (\text{Re}_d \text{Pr})^{1/3} \left( \frac{d}{L} \right)^{1/3} \left( \frac{\mu}{\mu_w} \right)^{0.14} \quad \text{Re}_d \text{Pr} \frac{d}{L} > 10
\]

All fluid properties are evaluated at the mean bulk temperature of the fluid, except \( \mu_w \), which is evaluated at the wall temperature.

6- For rough tubes flow, it is recommended to use the Colburn analogy relation to find the value of heat transfer coefficient

\[
\text{St}_b \frac{\text{Pr}_{f}}{2^{3/2}} = \text{Jf}
\]

\( \text{St}_b \) = Stanton numbe based on bulk temperature.  
\( \text{Pr}_{f} \) = Prandtl numbers based on film temperature, where \( T_f = (T_b + T_w)/2 \)

7- There is a more accurate equation can be used instead of last equation (in point 6) for fully developed turbulent flow in smooth tubes

\[
\text{Nu}_d = \frac{\text{Jf} \text{Re}_d \text{Pr}}{1.07 + 12.7(\text{Jf})^{1/2}(\text{Pr}_{f}^{2/3} - 1)} \left( \frac{\mu_b}{\mu_w} \right)^n
\]
This equation is applicable for the following ranges:

0.5 < Pr < 200 for 6 percent accuracy
0.5 < Pr < 2000 for 10 percent accuracy
10^4 < Re_d < 5 \times 10^6
0.8 < \mu_b/\mu_w < 40

Where n = 0.11 for Tw > Tb, n = 0.25 for Tw < Tb, and n = 0 for constant heat flux or for gases. All properties are evaluated at Tf = (Tw + Tb)/2 except for \mu_b and \mu_w.

**Note:** If the channel through which the fluid flows is not of circular cross section, it is recommended that the heat-transfer correlations be based on the hydraulic diameter DH, defined by

\[ D_H = \frac{4A}{P} \]

Where A is the cross-sectional area of the flow and P is the wetted perimeter. For example: for rectangular duct

\[ D_H = \frac{4[L \times W]}{[L + W] \times 2} \]

The hydraulic diameter should be used in calculating the Nusselt and Reynolds numbers, and in establishing the friction coefficient for use with the Reynolds analogy.

In general, the heat balance equation in such cases can be written as

\[ q = mcp(T_b_2 - T_b_1) = hAs(T_w - T_{bm}) \]

\[ T_{bm} = (T_b_1 + T_b_2)/2, \quad As = \pi DL, \quad m = \rho u A, \quad A = \pi D/4 \]